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Comparator Design: User-Defined Threshold With Asymmetrical Hysteresis

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When configuring a comparator circuit, it's common to add hysteresis to the threshold to provide noise immunity to the circuit. So rather than switching output states any time the input passes a single threshold value, the comparator switches states at two different values—a "high" threshold value when the output is low and a "low" threshold value when the output is high. Typically, these two threshold values are determined by setting a single threshold value with a single hysteresis value, so that in effect, the high and low thresholds are equidistant from the user-set threshold value. We'll call this usual case, a comparator threshold with symmetrical hysteresis.

While that may be fine for achieving noise immunity, there are cases where we'd like the thresholds to be set further apart. In this case, we'd like to be able to configure a comparator for a threshold with asymmetrical hysteresis. For example, being able to set the thresholds this way is convenient for providing a reliable safety feature in various power supplies incorporating voltage and current protection. Asymmetrical hysteresis provides for a speedy reaction at overload, while at the same time, allowing components to reliably cool down upon removal of the overload.

This article presents a comparator circuit that can be used to implement asymmetrical hysteresis and analyzes this circuit to obtain the formulas required to set the threshold value and the two values of hysteresis. The proposed schematic can utilize any modern comparator IC, having a push-pull output. (An open collector or drain output may also be used but will require some additional considerations to be implemented by the reader. Such considerations are beyond the scope of this article.) The proposed comparator circuit is shown in Fig. 1.

Our goal in this design is to define the values of all the resistors shown in the proposed circuit, in particular to determine the resistor values required to obtain the desired tripping points (i.e. the high and low thresholds). This requires an analysis of the operation of the proposed circuit.

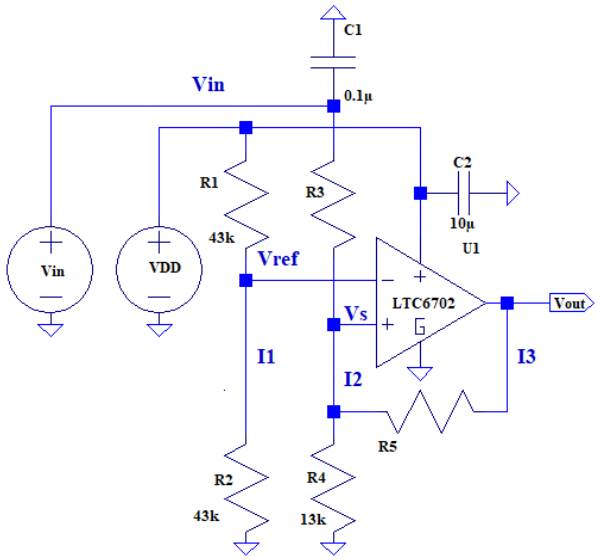

Fig. 1. A comparator circuit for achieving a user-defined threshold with asymmetrical hysteresis (i.e. independently set high and low thresholds).

Background Information And Definition of Terms

But before starting the design, we have to make a few assumptions that will direct and help the analysis. First of all, since the comparator effectively has two thresholds (a threshold plus/minus hysteresis) and two output states, we will have to review each of these output states and do the analysis accordingly. Secondly, in choosing the divider currents, their values must not affect the comparator operation. Thus the divider currents should be at least 2000 times higher than the comparator input current.

Also, to keep the comparator performance accurate, all resistors should have tolerances of 1% or better. Finally, for the sake of convenience, the comparator IC should be a readily available general-purpose part of adequate accuracy.

There are also some assumptions we can make about the internal operation of the comparator to simplify the analysis of the circuit. For instance, V_{out_max} (the value of the comparator output when high) is equal to the VDD value, while V_{out_min} (the value of the comparator output when low) is equal to 0. We can say this since we assume the internal MOSFETs' $R_{DS(ON)}$ is much lower than external resistors values. In addition, the comparator IC input bias voltage is negligible, and is overcome by the input voltage V_{in}.

Next, let's designate the following circuit parameters:

- V_{ref} is the reference voltage that defines the comparator threshold V_{th}.
- V_s is the voltage on the non-inverting input of the comparator that moves with respect to the threshold and should create hysteresis ΔV_{hyst} in the input voltage V_{in}.
- V_{out} is the comparator output voltage. Its minimum value is denoted by min, and maximum value is denoted by max (as noted above).
- ΔV_{hyst} is the hysteresis voltage that can be arbitrarily selected and have two values: positive (1) and negative (2) with respect to the threshold voltage.
- I1, I2, I3 are the currents in the respective schematic branches.
- V_{in1} is the minimum value of the input voltage when the comparator flips.
- V_{in2} is the maximum value of the input voltage when the comparator flips back.
- V_{th} is the median value of the threshold voltage, given by the customer.
- VDD1 is the IC power supply, which can be used for deriving V_{ref} if it is stable enough.

Analyzing $V_{out} = V_{out\_max}$

Fig. 2 shows the case where comparator output is high and V_{out} is at its max value, VDD1.

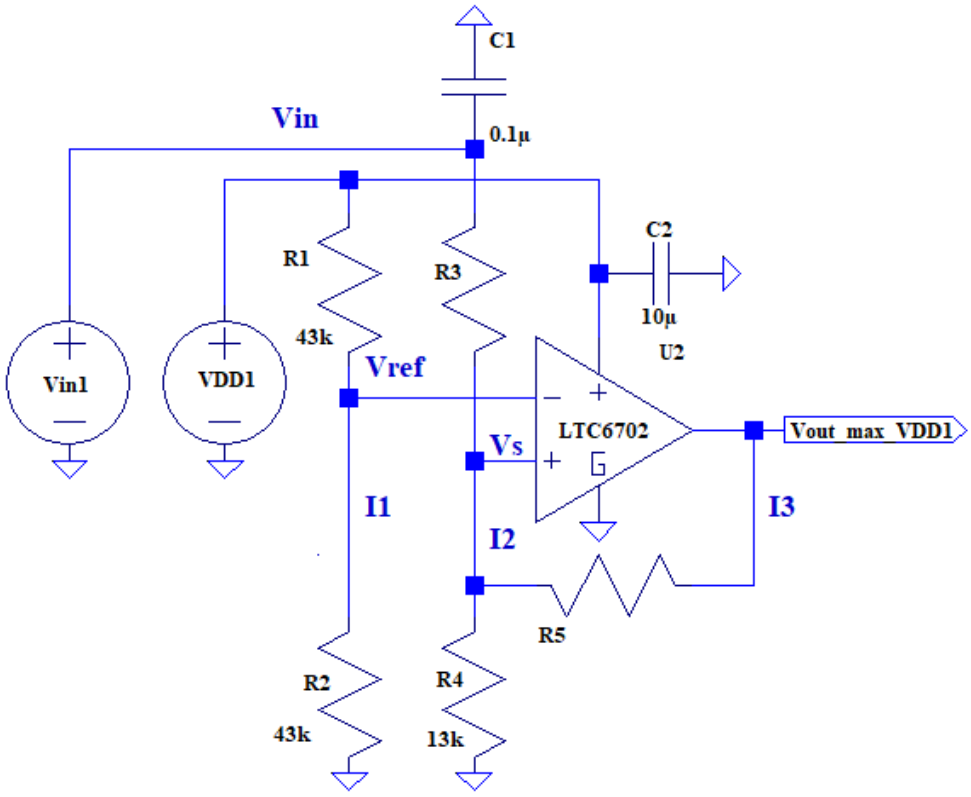

Fig. 2. Comparator output is high. Input voltage goes down.

This means V_{in1} is high enough to keep V_s higher than V_{ref}: When V_{in1} goes down to the lower tripping point that is V_{th} - ΔV_{hyst2}, the output V_{out} flips from V_{out_max} to V_{out_min}. It will flip back to V_{out_max} when the input voltage V_{in} hits V_{th} + ΔV_{hyst1}.

$$V_{in1} = V_{th} - \Delta V_{hyst2} \quad (1)$$

Using the loop-current method, we can write

$$V_{in1} = I_2 \cdot (R3 + R4) + I_3 \cdot R4 \quad (2)$$

$$VDD1 = I_3 \cdot (R5 + R4) + I_2 \cdot R4 \quad (3)$$

$$V_S = R4 \cdot (I_3 + I_2) \quad (4)$$

Per one of the assumptions

$$V_S = V_{ref} \quad (5)$$

Plugging in equation (1) into (2), we get

$$V_{th} - \Delta V_{hyst2} = I_2 \cdot (R3 + R4) + I_3 \cdot R4$$

Plugging equation (5) into (4), we obtain

$$V_{ref} = R4 \cdot (I_3 + I_2) \quad (6)$$

Analyzing $V_{out} = V_{out\_min}$

Fig. 3 shows the case where the comparator output is low and V_{out} is at its minimum value, O V.

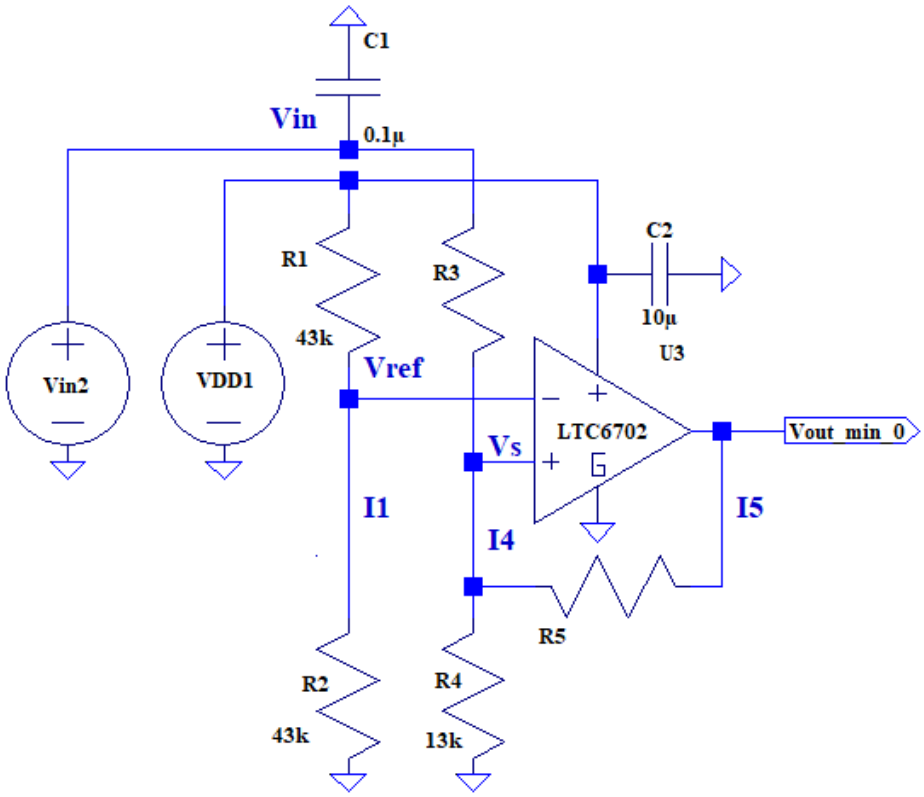

Fig. 3. Comparator output is low. Input voltage goes up.

When V_{in1} goes up to the tripping point that is $V_{th} + \Delta V_{hyst1}$

$$V_{in2} = V_{th} + \Delta V_{hyst1} \quad (7)$$

From Fig. 3, we can define V_{in2} in terms of I4:

$$V_{in2} = I_4 \cdot \left(R3 + \frac{R4 \cdot R5}{R4+R5}\right) \quad (8)$$

From both (7) and (8) we get:

$$V_{th} + \Delta V_{hyst1} = I_4 \cdot \left(R3 + \frac{R4 \cdot R5}{R4+R5}\right) \quad (9)$$

Since $V_s = V_{ref}$, we get:

$$V_{ref} = I_4 \cdot \frac{R4 \cdot R5}{R4+R5} \quad (10)$$

From equations (1), (2), (3), (6), (9) and (10) we can compose a system of equations to solve with respect to R3 and R5 using the following MathCAD-15 routine:

Given

$$V_{th} - \Delta V_{hyst2} = I_2 \cdot (R3 + R4) + I_3 \cdot R4$$

$$VDD1 = I_3 \cdot (R5 + R4) + I_2 \cdot R4$$

$$V_{ref} = R4 \cdot (I_3 + I_2)$$

$$V_{th} + \Delta V_{hyst1} = I_4 \cdot \left(R3 + \frac{R4 \cdot R5}{R4 + R5} \right)$$

$$V_{ref} = I_4 \cdot \frac{R4 \cdot R5}{R4 + R5}$$

$$\text{Find}(R3, R5, I_2, I_3, I_4) \rightarrow \begin{pmatrix} -\dfrac{R4 \cdot V_{ref} - R4 \cdot V_{th} + R4 \cdot \Delta V_{hyst2}}{VDD1} & -\dfrac{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}}{VDD1 \cdot V_{ref}} \\ -R4 & -\dfrac{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}}{V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2}} \\ \dfrac{VDD1}{R4} & \dfrac{VDD1 \cdot V_{ref}^2 + VDD1 \cdot V_{ref} \cdot \Delta V_{hyst2} - VDD1 \cdot V_{th} \cdot V_{ref}}{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}} \\ -\dfrac{VDD1 - V_{ref}}{R4} & \dfrac{V_{ref}^2 \cdot \Delta V_{hyst1} + V_{ref}^2 \cdot \Delta V_{hyst2} - VDD1 \cdot V_{ref} \cdot \Delta V_{hyst1} - VDD1 \cdot V_{ref} \cdot \Delta V_{hyst2}}{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}} \\ 0 & -\dfrac{VDD1 \cdot V_{ref} \cdot \Delta V_{hyst1} - VDD1 \cdot V_{ref}^2 + VDD1 \cdot V_{th} \cdot V_{ref}}{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}} \end{pmatrix} \quad (11)$$

From matrix (11):

$$R3 = -\frac{R4 \cdot (VDD1 \cdot V_{ref} - VDD1 \cdot V_{th} - VDD1 \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2})}{VDD1 \cdot V_{ref}} \quad (12)$$

$$R5 = -\frac{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}}{V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2}} \quad (13)$$

For the IC input leakage current, the LTC6702C datasheet specifies minimum, typical and maximum values of 0.001 nA, 1 nA, and 10 nA.

Taking the worst case

I_{in_leak} = 10 nA

and

VDD1 = 5 V

we can define the divider currents and select resistor values corresponding to these currents:

$I_{1\_min} = 2000 \cdot I_{in\_leak} = 2 \times 10^{-5}$ A

$$R1 + R2 = \frac{VDD1}{I_{1\_min}} \quad (14)$$

For simplicity, let's assume

R1 = R2 (15)

Then

$$R1 = \frac{VDD1}{2 \cdot I_{1\_min}} = 125 \times 10^3 \Omega$$

For noise reduction it is reasonable to use

R1 = 43 kΩ

R2 = R1 = 43 x 10^3 Ω

$$V_{ref} = \frac{VDD1}{2} = 2.5\ V$$

$$R4 = \frac{V_{ref}}{2000 \cdot I_{in\_leak}} = 125 \times 10^3 \Omega$$

For R4, we can choose a real value of

R4 = 13 kΩ

Low values for R1 and R4 keep R3 and R5 low enough, which is reasonable for reduction of noise pick-up.

If we assign physical values to the threshold and hysteresis levels,

V_{th} = 10 V

ΔV_{hyst2} = 0.6 V

ΔV_{hyst1} = 0.4 V

and define physical values for R3 and R5:

$$R3 = -\frac{R4 \cdot (VDD1 \cdot V_{ref} - VDD1 \cdot V_{th} - VDD1 \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2})}{VDD1 \cdot V_{ref}} = 38.48 \times 10^3 \Omega$$

$$R5 = -\frac{R4 \cdot V_{ref} \cdot \Delta V_{hyst1} - R4 \cdot VDD1 \cdot \Delta V_{hyst1} + R4 \cdot V_{ref} \cdot \Delta V_{hyst2} - R4 \cdot VDD1 \cdot V_{th} + R4 \cdot VDD1 \cdot V_{ref}}{V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2}} = 192.4 \text{ x } 10^3$$

These calculations can be easily adapted for EXCEL, which will facilitate comparison of diverse comparator designs with different values of threshold and hysteresis.

Worst-Case Analysis

The available resistor values will definitely differ from those calculated above. These resistors will also have specific tolerances, temperature walkaways (due to temperature coefficient of resistance) and value changes due to aging. These effects will influence the comparator accuracy and can even make it unusable in some conditions. Therefore it is reasonable to analyze how the threshold voltage would behave in the worst case when the resistors' values swing between their extremes.

Consider formula (12):

$$R3 = -\frac{R4 \cdot \left(VDD1 \cdot V_{ref} - VDD1 \cdot V_{th} - VDD1 \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2}\right)}{VDD1 \cdot V_{ref}}$$

Now find V_{th}:

$$V_{th} = \frac{V_{ref} \cdot \left[R3 + \frac{R4 \cdot \left(VDD1 \cdot V_{ref} - VDD1 \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2}\right)}{VDD1 \cdot V_{ref}}\right]}{R4}$$

In order to make computations concise, the following method of parameter notation and resistance acquisition is proposed. Here we define resistance as a function of a few parameters:

$\delta_{Rtol} = 0.01$

which is the tolerance provided by the resistor manufacturer. It may have both plus and minus signs.

$\delta_{Rth} = 10^{-4}/K$

which is the thermal coefficient. The sign is never specified by manufacturer.

$$\delta_{Rage} = \begin{pmatrix} 0 \\ 0.02 \end{pmatrix}$$

which is the aging coefficient and may have a positive sign only. The minimum value is 0, which corresponds to a new product.

T_{max} - T_{nom}

This is the largest temperature range to use. We don't specify values at minimum or maximum temperatures, just specify extremes. We'll assume the following values:

$T_{max} = 125$ K

$T_{nom} = 25$ K

Given the above terms, we can calculate the variation in any resistor value using the following general formula:

$$\mathrm{Rn}\left(\mathrm{Rx}, \delta_{\mathrm{Rtol}}, \delta_{\mathrm{Rth}}, \delta_{\mathrm{Rage}}, \mathrm{T_{nom}}, \mathrm{T_{max}}\right) = \mathrm{Rx}\begin{bmatrix}(1-\delta_{\mathrm{Rtol}})\cdot[1-(\mathrm{T_{max}}-\mathrm{T_{nom}})\cdot\delta_{\mathrm{Rth}}]\cdot\left(1+\delta_{\mathrm{Rage_0}}\right)\\(1+\delta_{\mathrm{Rtol}})\cdot[1+(\mathrm{T_{max}}-\mathrm{T_{nom}})\cdot\delta_{\mathrm{Rth}}]\cdot\left(1+\delta_{\mathrm{Rage_1}}\right)\end{bmatrix}$$

This is a general formula allowing us to calculate any resistor value including deviation within the same walkaway parameters.

As an example, let's apply the above equation to R3. In order to defining a resistor value matrix, it is necessary to put down the Rx value, which is the nominal value. For R3, we'll choose

Rx = 38.3 kΩ

Then

$$\mathrm{R3} = \mathrm{Rn}\left(\mathrm{Rx}, \delta_{\mathrm{Rtol}}, \delta_{\mathrm{Rth}}, \delta_{\mathrm{Rage}}, \mathrm{T_{nom}}, \mathrm{T_{max}}\right) = \begin{pmatrix}37.538\ \mathrm{x}\ 10^3\\38.3\ \mathrm{x}\ 10^3\\39.851\ \mathrm{x}\ 10^3\end{pmatrix}\Omega$$

For R5

Rx = 191 kΩ

Therefore

$$\mathrm{R5} = \mathrm{Rn}\left(\mathrm{Rx}, \delta_{\mathrm{Rtol}}, \delta_{\mathrm{Rth}}, \delta_{\mathrm{Rage}}, \mathrm{T_{nom}}, \mathrm{T_{max}}\right) = \begin{pmatrix}187.199\ \mathrm{x}\ 10^3\\199\ \mathrm{x}\ 10^3\\198.736\ \mathrm{x}\ 10^3\end{pmatrix}\Omega$$

For R1

Rx = 43 kΩ

$$\mathrm{R1} = \mathrm{Rn}\left(\mathrm{Rx}, \delta_{\mathrm{Rtol}}, \delta_{\mathrm{Rth}}, \delta_{\mathrm{Rage}}, \mathrm{T_{nom}}, \mathrm{T_{max}}\right) = \begin{pmatrix}42.144\ \mathrm{x}\ 10^3\\43\ \mathrm{x}\ 10^3\\44.742\ \mathrm{x}\ 10^3\end{pmatrix}\Omega$$

Then

$$\mathrm{R2} = \mathrm{reverse}(\mathrm{R1}) = \begin{pmatrix}44.742\ \mathrm{x}\ 10^3\\43\ \mathrm{x}\ 10^3\\42.144\ \mathrm{x}\ 10^3\end{pmatrix}\Omega$$

Given the following variations in VDD1 and Vref

$$\mathrm{VDD1} = 5\cdot\begin{pmatrix}1.01\\1\\0.99\end{pmatrix}\mathrm{V}$$

$$V_{ref} = \overrightarrow{\left(\frac{VDD1}{R1 + R2} \cdot R2\right)} = \begin{pmatrix} 2.6 \\ 2.5 \\ 2.401 \end{pmatrix} V$$

the threshold voltage is

$$V_{th} = \frac{\overrightarrow{V_{ref} \cdot \left[R3 + \frac{R4 \cdot \left(VDD1 \cdot V_{ref} - VDD1 \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst1} + V_{ref} \cdot \Delta V_{hyst2}\right)}{VDD1 \cdot V_{ref}}\right]}}{R4} = \begin{pmatrix} 10.377 \\ 9.965 \\ 9.56 \end{pmatrix} V$$

Defining the nominal threshold voltage as V_{th0}

V_{th0} = 10 V

we can define the RMS deviation as

$$\sigma = \sqrt{\frac{(V_{th0} - V_{th_0})^2 + (V_{th0} - V_{th_1})^2 + (V_{th0} - V_{th_2})^2}{3}} = 0.335 \text{ V}$$

which is a less than a 3.5% error:

$$\frac{\sigma}{V_{th0}} = 0.034$$

Defining asymmetrical thresholds allows for designing versatile and accurate comparators for power supplies. The degree of error should be sufficient for maintaining the "stiff" threshold and tripping points (V.th ±ΔV.hyst1) required for overload protection in most power supply applications.

Reference

"Comparator with Hysteresis Reference Design" by Art Kay and Timothy Claycomb, Texas Instruments.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

For further reading on designing circuit protection in power supplies, see the How2Power Design Guide, locate the Design Area category and select "Power Protection".