Misconceptions In Power Magnetics

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

Magnetic components appear to be so simple—just two parts, a core and some wire wrapped around it. How could that be very complicated? If you ask this question of yourself seriously enough, you begin your own descent into the abyss of magnetics design.

As a "recovering magnetaholic", I have learned that magnetics really is simple, but the path to simplicity has some misleading ideas and some that are not actually true, though they are widespread. More importantly, some basic concepts that should be widely known are not. This article is a chat about some of them.

A Design Project Generates Unanswered Questions

A few years ago, I began design work on a battery converter for an off-grid inverter. The converter has low-input-resistance (low V, high I), from a 24-V battery and at 1 kW, by Watt's Law, takes in about 42 A. After some research, I concluded that the best power circuit for this application is that of a *boost push-pull* (BPP), a common-active PWM-switch converter with a transformer, shown in Fig. 1.

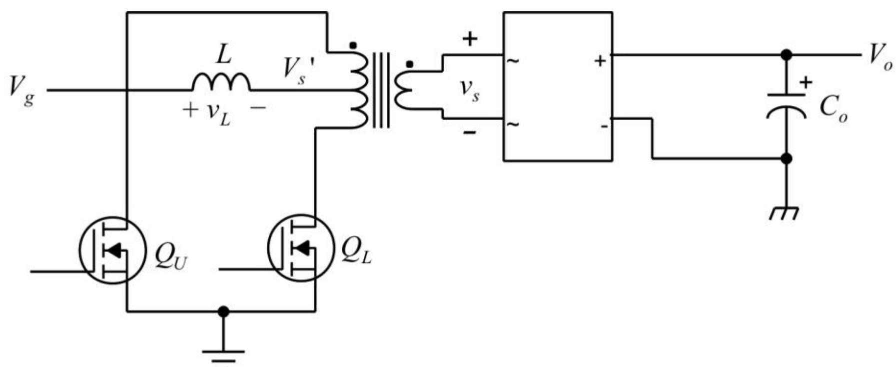

Fig 1. A boost push-pull (BPP) converter, the optimal choice for low-R_g (low input voltage, high current) conversion.

The converter has an input inductor, L, and a center-tapped transformer, giving it essentially two primary windings and a 170-V secondary winding. The secondary voltage referred to the primary appears at the center-tap as $V_s' = n \cdot V_s$ where the turns ratio, $n = N_p/N_s = 1/5$—a "step-up" transformer. Then $V_s' = 34$ V, and is above the maximum specified input voltage, V_g, of 30 V.

The amount of power that the inductor must be capable of transferring decreases as V_s' decreases, which reduces the inductor voltage, v_L, and flux change, $\Delta\lambda$, and thus its transfer power. However, unless $V_s' > V_g$, the converter quits boosting and the direction of power flow reverses. With changes in n, inductor power is greatly affected but not so much for transformer power because all power transferred between input and output ports must go through it.

The circuit poses questions about the design of the inductor and transformer and how to optimize them. Seemingly reasonable questions have not been given convincing answers. Textbooks do not give them because they do not address design *optimization* as they concentrate primarily on the science of magnetics in its application to engineering. Engineering, however, is not only about science but is distinctively about design, optimization, and about achieving goals that solve human physical problems.

Maximum-Power Misconception

One of the semi-myths of magnetics design that I had carried in my notebook and mind for many years is the belief that maximum power is transferred through a transformer when the winding and core losses are equal, or average $P_w = P_c$. This is usually demonstrated by using the *maximum power-transfer theorem*, by letting winding loss be included in a series winding resistance with a shunt resistance for core loss, all referred to the secondary winding, as shown in Fig. 2.

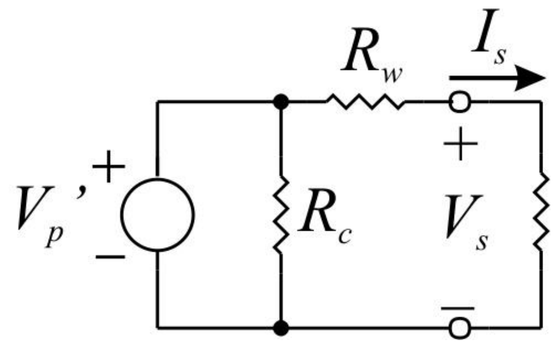

Fig. 2. Interwinding power transfer model, referred to the secondary winding with a resistive load, secondary (load) current of I_s and voltage of V_s. R_c is the secondary-referred core resistance representing core power loss, and R_w is the secondary-referred winding resistance of both windings combined.

The core-loss resistance, R_c is across the secondary-referred primary winding voltage, V_p'. Because core loss varies directly with flux change, which is proportional to voltage, core loss can be modeled with R_c. The dictum of maximum power transfer under the condition, $P_w = P_c$ is true only at zero power, or 100% efficiency. Otherwise, maximum power is transferred—that is, maximum efficiency occurs—when P_w is greater than P_c, slightly at high efficiency and more so at lower efficiency.

The primary winding is driven by a constant-voltage source, referred to the secondary as V_p' with a secondary-winding terminal voltage of V_s. The secondary output power is

$$P_s = V_s \cdot I_s \;,\; I_s = \frac{V_p' - V_s}{R_w}$$

As R_w decreases, P_s increases along with V_s and I_s. P_c is a function of V_p' and P_w is a function of both V_p' and V_s. The core power loss, P_c and winding power loss, P_w are

$$P_c = \frac{V_p'^2}{R_c} \;;\; P_w = \frac{(V_p' - V_s)^2}{R_w}$$

The total transformer power loss is

$$P_t = P_c + P_w = \frac{V_p'^2}{R_c} + \frac{(V_p' - V_s)^2}{R_w} = \frac{V_p'^2}{R_c} + I_s^2 \cdot R_w$$

The primary-winding input power is

$$P_p = P_s + P_t$$

Power transfer as an output/input ratio is the same as *efficiency*,

$$\eta = \frac{P_s}{P_p} = \frac{P_s}{P_s + P_t} = \frac{V_s \cdot I_s}{V_s \cdot I_s + P_c + I_s^2 \cdot R_w}$$

The plots for the circuit are shown in Fig. 3 for a constant V_p' = 10 V, R_c = 9 Ω, and R_w = 1 Ω. The power quantities are in W and η is in percentage (%). P_c and P_w cross at 3.33 A while η peaks at 2.4 A with η = 51.9% and where $P_c = P_w$, $\eta < \eta_{max}$ by about 1.9%.

The crux of the error is that the maximum power-transfer theorem from introductory passive-circuits courses is applied. It is derived with a voltage source having a series resistance connected to a resistive load. The error is that the theorem does not apply because R_c is not included in its circuit model.

What is needed instead is a *maximum output-power theorem*. This is a more difficult case (see the "General Power-Transfer Circuit Model" in reference 1). If core loss is negligible for high-η design, then $P_c = P_w$ is approximately correct. However, to believe that it has been derived as exact is misleading.

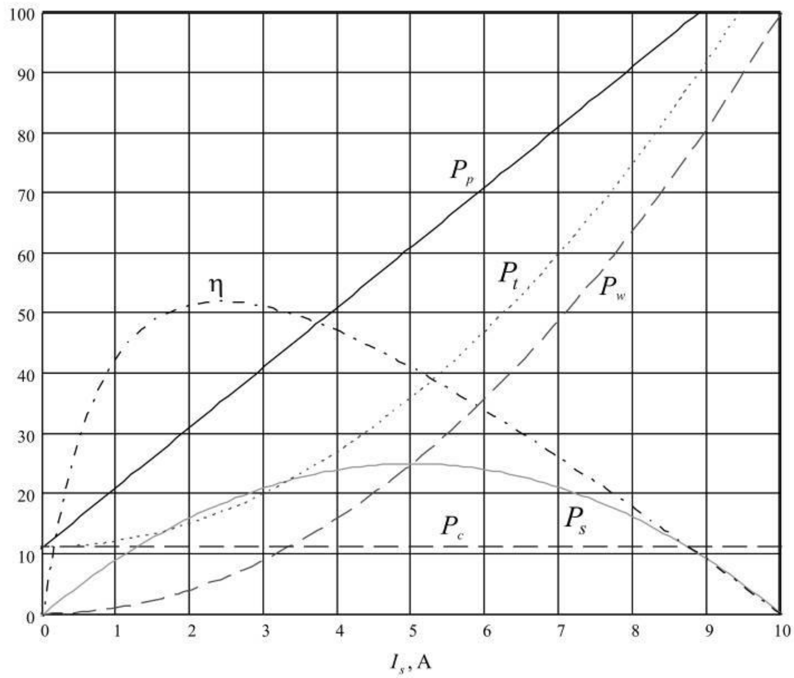

Fig. 3. Graph showing for the example given that maximum power is not transferred when core and winding losses are equal. Winding loss, P_w and core loss, P_c cross at two values of I_s, neither of which is at peak efficiency, η.

There is a subtle calculus error in the commonplace derivation of maximum η (see "Maximum Power Transfer" in reference 1). Hence, one longstanding proposition found in much of the power-electronics literature is not really true. For high efficiency, it is approximately true, which is why it has been possible to promulgate it all these years without undue suspicion.

Yet if you suppose it is *always* true, and if you are concerned about the efficiency of your converter design over its full input-current range, you find the efficiency falls off rather abruptly at low current. And here the old dictum begins to fail noticeably.

Optimal Waveshape For A Core Material

Another underappreciated loose end is the relationship between circuit waveforms and optimal core material. Seemingly, there should be no direct relationship, but there is.[4]

The question of how to select the right core material for a given magnetics design starts with the basic limitations on cores. Frequencies much above audio eliminate 60-Hz transformer cores such as 3% silicon steel. All of the low-frequency materials have excessive hysteresis loss for operation at power converter switching frequencies.

Operation at the highest possible frequency maximizes power-transfer density because power transfer in a converter is directly proportional to it. Frequency is also related to the more basic core limitation of power loss and maximum allowable core temperature.

The other basic core limitation is saturation, in which its magnetic properties diminish. Power loss limits a combination of the frequency and the amplitude of the core field density ripple, $\hat{B}_{\sim} = \Delta B/2$. Saturation limits the static field intensity, $\overline{H}$, and the two combined limit the *ripple factor*, γ. In field quantities,

$$\gamma = \frac{\hat{i}_{\sim}}{\bar{i}} = \frac{\hat{\phi}_{\sim} / \mathcal{L}}{\overline{H} \cdot l} = \frac{\hat{B}_{\sim} \cdot A / \mathcal{L}}{(\overline{B} / \mu) \cdot l} = \frac{\hat{B}_{\sim}}{\overline{B}}$$

where $\mathcal{L} = \mu \cdot A / l$ = field inductance, A = magnetic cross-sectional area and l = magnetic path length.

The ripple factor of the winding current waveform is determined by core loss in the numerator and saturation in the denominator. Substitute the maximum allowable values for a given core material, and γ_{opt} for that particular material results. This γ is optimum because it allows maximum energy transfer density, Δw through the core, by achieving maximum ΔB and $\overline{H}$ for which

$$\Delta w = \Delta B \cdot \overline{H}$$

Ripple factor is a significant core material performance parameter which, to my knowledge, has not been made explicit in the development of power magnetics. Magnetics suppliers do not include it in core specifications. Sanjaya Maniktala has noted it[2] in his books, but in general most engineers seem unaware of the connection between ripple factor and core material optimization for power transfer.

Winding Area Allotment

Another possible misconception that relates to both converter circuit and transformer design is how much area should optimally be allotted to each winding. Usually, there is nothing mysterious about this; the primary and secondary windings should each be allotted equal area. The rationale is simple. Power is being transferred from primary to secondary winding and except for the loss of a little power as heating rate in the transfer, they are equal.

Transformer thermal design is optimized by having no part of the transformer hotter than any other part. Although this is an ideal, it is approached by designing for equal power-loss density. To achieve this for windings of near-equal power, their areas should also be made equal.

Where the windings are wound sequentially, the primary is wound first for highest coupling and power transfer to the core, and it is thermally the farthest from ambient. As a result of that factor, for sequential winding design, the primary winding might be given somewhat larger area to reduce its power-loss density and hence its temperature.

However, the heat path of the primary is almost completely through the metal of the secondary winding. This heats the secondary with both its own loss and that of the primary winding. Therefore, a thermal optimization lowers the power density of the secondary by giving it more area. Generally though, allotting equal areas for primary and secondary windings balances these tradeoffs and is considered optimal. For unibundle or other multifilar winding configurations, where the windings are wound simultaneously, the equal-area guideline might also apply.

A thunderclap is now heard in this otherwise placid scenario when the boost push-pull (BPP) converter is considered. It has different primary and secondary winding waveshapes. The complication is that it has (functionally) two identical primary windings and one secondary winding.

The primary windings alternate in conduction cycles while the secondary conducts each cycle. The fractional allotment of total core window area, A_w of a winding is k_{ww}: $A_{wwp} = k_{wwp} \cdot A_w$; $A_{wws} = k_{wws} \cdot A_w$. Then the area ratio of primary to secondary windings for the BPP is

$$Y = \frac{A_{wwp}}{A_{wws}} = \frac{2 \cdot \tilde{i}_p}{\tilde{i}_s / n} = n \cdot \sqrt{D' \cdot (1 + D')} \cdot \frac{V_s}{V_g} = \sqrt{D' \cdot (1 + D')} \cdot \frac{V_s'}{V_g}, \; V_s = 170 \text{ V} ; 1/n = 5 ; V_s' = n \cdot V_s = 34.0 \text{ V}$$

In most converter transformers, the winding area ratio is optimally $Y = 1$. In the Volksinverter series of articles, which begins running in this issue of How2Power Today,[5] the waveform equations are derived, but here they are given. The primary and secondary currents are $\tilde{i}_p$ and $\tilde{i}_s$, converter PWM duty-ratio = D, $D' = 1 - D$, n = transformer turns ratio of either primary winding to the secondary winding, V_s = secondary voltage amplitude,

V_g = converter input voltage, and V_S' is the secondary voltage, referred to the primary and probed at the center-tap of the two primary windings. Some math gives the basic area relationships:

$$A_w = A_{wwp} + A_{wws} \Rightarrow A_{wwp} = A_w - A_{wws}\ ;\ \frac{A_{wws}}{A_w} = \frac{1}{Y+1}\ ;\ \frac{A_{wwp}}{A_w} = \frac{Y}{Y+1}$$

The Volksinverter specified input voltage range is 20 V to 30 V. Over this range, the optimal Y changes value, but as a transformer, n and Y and the other parameters are fixed in design. Our challenge is picking the best compromise optimum from the following table of values.

Table. Optimizing the allotment of winding area to primary and secondary windings.

V_g, V	D'	Y	$\frac{A_{wws}}{A_w}$	$\frac{A_{wwp}}{A_w}$
20	0.588	1.643	0.378	0.622
25	0.735	1.536	0.394	0.606
30	0.882	1.460	0.4065	0.5935

The chosen option allots $(0.6){\cdot}A_w$ to the primary windings, with $(0.3){\cdot}A_w$ to each, and $(0.4){\cdot}A_w$ to the secondary winding, as shown in Fig. 4.

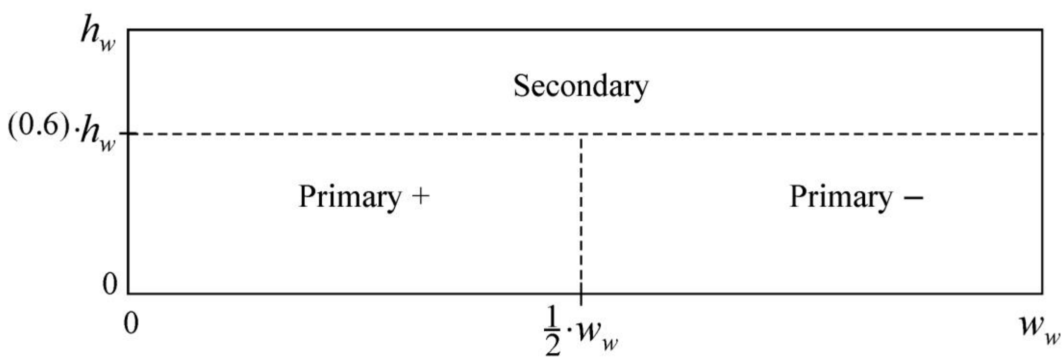

Fig. 4. The selected winding-area allotment for transformer primaries and secondary in the boost push-pull converter. The primary windings have k_{wwp} = 0.3 and the secondary winding has k_{wws} = 0.4. The primary winding is optimal at more than half the window area.

Optimal Core Shapes

Robert Jensen and his advisor at Dartmouth College, Charles Sullivan, did an empirical study of core shapes that minimize thermal resistance. A geometric optimization of core shape involves the winding window aspect ratio of width (the dimension along which winding turns proceed) to height, which affects the number of layers, and is optimally between 1 and 2 for minimum P_w when dynamic resistance is taken into account.[3]

Commercial core shapes, including EE, ETD (4 to 5), EC, RM (2 to 3) and PQ (1.5 to 3), are in the range of 2 to 5 and typically around 3, resulting in about a third greater loss than optimal. Although a large width/height aspect ratio reduces the number of winding layers and eddy-current resistance ratio, F_R, it also increases the winding length for a given window relative to the window area, thus increasing winding resistance.

RM and PQ shapes, which offer the optimal winding window aspect ratios, are found in power inverters such as the Statpower 500, but the core shapes that the Jensen and Sullivan study found to be less thermally efficient are also popular and not relegated to obsolescence. Perhaps that will change as these findings diffuse into the broader power-electronics design world.

How To Avoid Misconceptions

Acquiring misleading ideas about magnetics can be avoided with a solid grounding in the fundamental principles that will be as true a century from now as a century ago. In surveying the literature, there are two textbooks and a research site that I found to be the most helpful, all from leading magnetics researchers.

The most rigorous and complete book I found on basic magnetics is *High-Frequency Magnetic Components, Second Edition*, Wiley, 2014, by Marian K. Kazimierczuk of Wright State U.[6] In this work, the author has worked out all the equations relating to the fundaments of component *analysis*, and some for design.

The second book is thinner and has a different emphasis to it, *Transformers and Inductors for Power Electronics: Theory, Design and Applications*, by W. Gerard Hurley in Galway, Ireland, and Werner Wölfle.[7] This text has slightly less math density and covers some different topics than Kazimierczuk's book.

And finally, from Charles Sullivan's website in the Thayer School of Engineering at Dartmouth College you can download his main research papers on magnetics.[8] Sullivan is primarily interested in how to minimize core and winding losses by reducing the eddy-current effects through wire strand twisting and braiding, and also in how core dimensions affect core loss.

These three sources provide a foundation for proceeding to magnetics design optimization in my book.[1]. If you want a PDF copy (the paper version sustains a printing cost), join the Innovatia Associates list on the Innovatia website.[4]

References

1. *Power Magnetics Design Optimization*, Dennis L. Feucht, Innovatia, innovatia.com, pp. 270 - 273.

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8. http://thayer.dartmouth.edu.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For more on magnetics design, see these How2Power Design Guide search results.