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Transformer Winding Design: Bundle Layer Approximation As Rings

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Wire *bundles* consist of *strands* of individual wire conductors bound together and wound as turns in a winding of a magnetic component such as a transformer. Having less than five strands minimizes the proximity effect within a bundle when twisted. Some bundles have many more than five strands, and within the bundle, those strands form the same kind of pattern as wire wound in layers in a winding, with the same packing properties.

With multiple layers of strands within a bundle, the proximity effect becomes significant. To estimate what it might be from Dowell's equation (or algebraic approximations of it) some estimate of the number of layers of strands, M_s, within the bundle is needed. An estimate of the number of layers is needed because the configuration of strands in a bundle does not mirror the planar layering of an unbundled winding. This circular geometry leads to different methods of counting, or calculating the number of layers. This article derives various formulas for estimating M_s and gives rationales for them.

Layer Approximation Based On Semi-Circles Of Rings

When strands are added to a bundle, they naturally compress into shapes of minimal cross-sectional area. For many strands, these shapes take the form of *rings* of strands, as shown in Fig. 1.

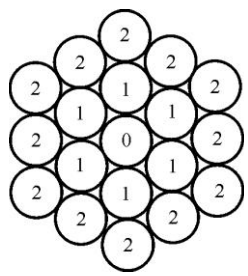

Fig. 1. Multiple strands in a wire bundle form rings. The number of strands per ring increases with ring number, q.

As strands are added to the bundle, they form rings with an increasing number of strands per ring. The center strand is layer 0. The first ring out from center, ring 1, forms the second layer and 6 strands complete it. The second ring has 12 strands, and for a complete ring, *q,* the number of strands in the bundle is

$$N_s(q) = N_s(q-1) + 6\cdot q = N_s(0) + \sum_{m=1}^{q} 6\cdot m = 1 + 6\cdot\frac{q\cdot(q+1)}{2} = 1 + 3\cdot q\cdot(q+1)\,,\; N_s(0) = 1$$

where the first term, $N_s(0) = 1$ includes the single strand of layer 0. $N_s(q)$ is solved in polynomial form as

$$q^2 + q - \frac{N_s - 1}{3} = 0 \;\Rightarrow\; q = \tfrac{1}{2}\cdot\left(\sqrt{1+\tfrac{4}{3}\cdot(N_s-1)} - 1\right)$$

The result is $q + 1$ layers, counting q from zero.

However, because of the closed curvature of the rings, strands of a ring are closer to each other in the ring than in a linear configuration, where they are at opposite ends of the linear segment. On each semi-circle, ring strands in each semi-circle function as a layer because they are no farther from each other than the ring diameter for strands everywhere around the ring. By this rationale, each semi-circle of a ring is a layer, and the number of layers is related to the number of rings by

$$M_{sq} = 2\cdot q + 1\,,\; q = 1, 2, \ldots \;\Rightarrow\; M_{sq} = \sqrt{1+\tfrac{4}{3}\cdot(N_s-1)}$$

where M_{sq} represents strand layers M_s counted according to the number of semi-circles for all rings (q), dependent on number of strands.

The bundle insulated radius in ring count, q is

$$r_{bw} = r_b + r_{cw} = (2 \cdot q + 1) \cdot r_{cw} = M_{sq} \cdot r_{cw}\,,\; q = 1, 2, \ldots$$

where r_b is the (untwisted, uncompressed, non-overlapping-layer) bundle radius to the center of the outer ring and r_{bw} is the bundle radius to its outer radius, an additional ring radius of strand radius, r_{cw}.

For a small number of strands—less than two complete rings, or $N_s < 7$—the estimates are not of much value; geometric bundle analysis is more exacting and for proximity-effect estimates, are not needed because the intrabundle effect is approximately zero for twisted wire with $N_s < 5$.

Layer Approximation By Bundle Radius

Another way to estimate layers in a bundle of strand rings is to draw circles through the centers of the strands in a given ring, q. The bundle radius, $r_b(q)$ is to the center of the strand of insulated wire radius, r_{cw}. The ring centers of aligned strands are separated by strand diameter, $2 \cdot r_{cw}$, and

$$r_b(q) = (2 \cdot r_{cw}) \cdot q\,,\; q = 0, \ldots \;\Rightarrow r_b = 0,\, 2 \cdot r_{cw},\, 4 \cdot r_{cw},\, \ldots$$

The number of strands that can be placed around the circumference of $2 \cdot \pi \cdot r_b(q)$ is

$$N_s = \frac{2 \cdot \pi \cdot r_b(q)}{2 \cdot r_{cw}} = 2 \cdot \pi \cdot q\,,\; q = 1, 2, \ldots$$

Only an integer number, N_s of strands is possible though this approximation for N_s can have a fractional value. The total calculated number of strands with an outer ring, q is

$$N_s(q) = 1 + \sum_{m=1}^{q} 2 \cdot \pi \cdot m = 1 + 2 \cdot \pi \cdot \left(\frac{q \cdot (q+1)}{2} \right) \Rightarrow q^2 + q - \frac{N_s - 1}{\pi} = 0 \;\Rightarrow\; q = \tfrac{1}{2} \cdot \left(\sqrt{1 + \tfrac{4}{\pi} \cdot (N_s - 1)} - 1 \right)$$

Applying the same rationale as before for the relationship of rings to layers,

$$M_{sb} = 2 \cdot q + 1\,,\; q = 1, 2, \ldots \;\Rightarrow\; M_{sb} = \sqrt{1 + \tfrac{4}{\pi} \cdot (N_s - 1)}$$

where M_{sb} represents layers counted according to the number sides for all rings (q) based on bundle radius (b) and dependent on number of strands.

M_{sb} and M_{sq} are tabulated below. The difference of 4/3 and $4/\pi$ causes M_{sb} and M_{sq} to diverge with increasing q.

Table 1. Calculating layers as a function of rings using the two geometric methods.

q	N_s	M_{sb}	M_{sq}
0	1	1	1
0+	4	2.20	2.24
1−	5	2.47	2.52
1	7	2.94	3
2	19	4.89	5
3	37	6.84	7
4	61	8.80	9
5	91	10.75	11

M_{sb} is based on circular rings whereas M_{sq} is based on adding $6 \cdot q$ to each successive ring instead of $2 \cdot \pi \cdot q$. Geometrically, it is the difference between arc and secant length, as shown in Fig. 2. The greater length of an arc than a secant causes $M_{sb} < M_{sq}$.

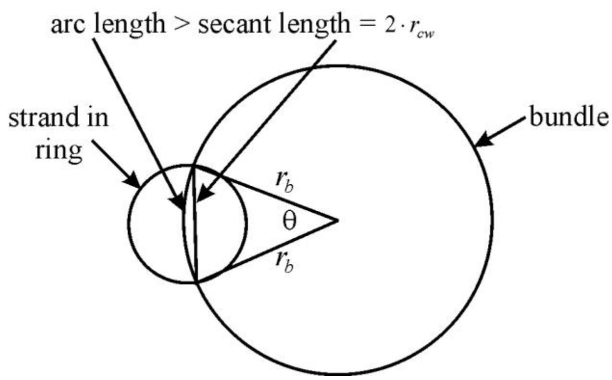

Fig. 2. M_{sq} approximation of M_{sb}. The difference is in the lengths of arc for M_{sb} and secant for M_{sq}.

Layer Approximation By Bundle And Strand Areas

The number of strand layers, M_s in a winding bundle can also be approximated by configuring the N_s strands of the bundle as a square with $N_s^{1/2}$ strands per side. The strand layers in the bundle are

$$M_s \approx M_{sh} = \sqrt{N_s}$$

where M_{sh} represents an area-based counting of layers based on a squaring of the bundle into $N_s^{1/2}$ strands (s) per side and allowing for hexagonally (h) packed strands.

The previous formulas for bundle layers based on rings ignore how layers pack together. They assume that the rings are packed with strand centers of rings aligned, or *square layering*, not hexagonal layering.

In contrast, for layer approximations based on area, bundle packing factor, k_{pb} is included in the derivation of formulas.[1] The conductive and insulated strand areas differ by the *porosity*, $k_{pw} = A_c/A_{cw}$, where A_{cw} is the strand area including insulation (and any other spacing between strands). The other factor is the fill factor, k_{pf} of the nonconductive spaces between round strands. For the bundle, $k_{pb} = k_{pw} \cdot k_{pf}$.

A k_{pf} value is derived from a square bundle from the definition of k_{pb};

$$k_{pb} = k_{pw} \cdot k_{pf} = \frac{N_s \cdot A_c}{A_{bw}} = \frac{N_s \cdot k_{pw} \cdot A_{cw}}{A_{bw}} \Rightarrow k_{pf} = \frac{N_s \cdot [\pi \cdot r_{cw}^2]}{[\sqrt{N_s} \cdot (2 \cdot r_{cw})]^2} = \frac{\pi}{4}$$

where A_c = strand conductive area, A_{cw} = insulated strand area, and A_{bw} = bundle area. The value of k_{pf} for round bundles is

$$k_{pf} = \frac{N_s \cdot A_{cw}}{A_{bw}} = N_s \cdot \frac{\pi \cdot r_{cw}^2}{\pi \cdot r_{bw}^2} = N_s \cdot \left(\frac{r_{cw}}{r_{bw}}\right)^2$$

A bundle-layer approximation is then defined by the bundle-to-strand radius ratio with hexagonal packing. Defining the area-based, round-bundle layers as M_{sr} and with $k_{pf} = \pi/4$,

$$M_s \approx M_{sr} = \frac{r_{bw}}{r_{cw}} = \sqrt{\frac{N_s}{k_{pf}}} \approx \sqrt{\frac{4}{\pi} \cdot N_s}$$

where M_{sr} represents an area-based counting of layers in a round bundle (r).

This ratio was given previously in rings as $M_s(q) = 2 \cdot q + 1$. Equating the new M_s approximation of M_{sr} to $M_s(q)$, and solving for q,

$$q_r = \frac{1}{2} \cdot \left(\sqrt{\frac{4}{\pi} \cdot N_s} - 1 \right)$$

The values of $q_r(N_s)$ are not exactly q as given in Table 2 but are close and are non-integer. Consequently, the area derivations of bundle layers results in the number of layers for a given N_s that are close to those resulting from ring-based derivations. The four approximations are compared in the following table.

Table 2. Comparing the different approximations of layers (M_s)
and the number of rings (q) associated with the different methods.

q	N_s	M_{sb}	M_{sq}	M_{sh}	M_{sr}	q_r
0	1	1	1	1	1.13	0.06
0+	4	2.20	2.24	2	2.26	0.63
1−	5	2.47	2.52	2.24	2.52	0.76
1	7	2.94	3	2.65	2.99	0.99
2	19	4.89	5	4.36	4.92	1.96
3	37	6.84	7	6.08	6.86	2.93
4	61	8.80	9	7.81	8.81	3.91
5	91	10.75	11	9.54	10.76	4.88

The equations for the table values are

$$M_{sb} = \sqrt{1 + \tfrac{4}{\pi} \cdot (N_s - 1)} \;;\; M_{sq} = \sqrt{1 + \tfrac{4}{3} \cdot (N_s - 1)} \;;\; M_{sh} = \sqrt{N_s} \;;\; M_{sr} = \sqrt{\frac{N_s}{k_{pf}}} \approx \sqrt{\frac{4}{\pi} \cdot N_s}$$

A somewhat different definition of M_{sb} removes k_{pb} from the formula for M_s. Instead of ratioing r_{bw} to r_{cw}, the packing factor is included in a modified r_{cw} that is effectively the strand radius with the packing effect on area included, or r_{cwp}. Then the bundle layers of *packed* radius[2] is defined as

$$M_s \approx M_{sb} = \frac{r_{bw}}{r_{cwp}} = \frac{r_{bw}}{r_{cw} / \sqrt{k_{pf}}} = \sqrt{N_s} = M_{sh} \;\;,\;\; r_{cwp} = \frac{r_{cw}}{\sqrt{k_{pf}}}$$

By this definition of M_s with k_{pf} taken into account, we see that the square-bundle M_{sh} implicitly includes packing. Whether strands are bundled into a square or a round form makes no difference in how they pack together, and the hex-layered k_{pf} is the same for both. Consequently, $M_{sb} = M_{sh}$ and $M_s \approx \sqrt{N_s}$. This also follows from equating square and round bundle areas;

$$A_{sq} = \left[\sqrt{N_s} \cdot (2 \cdot r_{cw})\right]^2 = N_s \cdot 4 \cdot r_{cw}^2 \;;\;\; A_r = \pi \cdot r_{bw}^2 \;\Rightarrow\; A_{sq} = A_r \;\Rightarrow\; M_{sr} = \frac{r_{bw}}{r_{cw}} = \sqrt{\frac{N_s}{k_{pf}}} = \frac{M_{sb}}{\sqrt{k_{pf}}} \;,\;\; k_{pf} = \frac{\pi}{4}$$

The above formulas and their derivations are based on bundles that are untwisted and uncompressed (round).[3] Twisted sub-bundles forming a larger Litz-wire bundle also can interpenetrate for small N_s because the twist-circle has gaps along its periphery where strands from adjacent sub-bundles can penetrate.[4]

References

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Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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