ISSUE: May 2022

Designing An Open-Source Power Inverter (Part 4): The Optimal Power-Line Waveshape

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

What is the optimal waveshape for a power-line inverter to generate? In part 2 of this series,[1-3] we began to address this question, noting the effects of inverter output waveshape on power component ratings, efficiency and its influence on the choice of the two-stage power architecture selected for this Volksinverter design. We concluded by noting that ultimately these considerations led to our selection of a third-harmonic sine wave (3HSW) for the inverter waveshape. In this part 4, we'll delve further into why that waveshape is optimal.

After comparing waveshape characteristics of sine waves, square waves and third-harmonic sine waves, we'll look at some PWM switching techniques that can be applied to the H-bridge to generate these waveforms. As was true in part 3 when discussing power-transfer circuit options, this discussion of inverter-output waveshapes is meant to help designers evaluate waveshapes for a range of inverter design requirements and not just the Volksinverter.

Square Waves Vs. Sine Waves

The most common waveshapes are bipolar square waves and sine waves. One of the difficulties in producing a sine-wave output is the power demands it places on components, which can be expressed as a waveform performance parameter.

The equations for a sine wave are

$$v = V \cdot \sin(\omega \cdot t)\,;\, i = I \cdot \sin(\omega \cdot t)$$

Then power,

$$P = v \cdot i = V \cdot I \cdot \sin^2(\omega \cdot t)$$

Average power is thus

$$\overline{P} = \frac{V \cdot I}{2} = \frac{\hat{P}}{2} \Rightarrow \frac{\hat{P}}{\overline{P}} = 2$$

For voltage or current, $x = X \cdot \sin(\omega \cdot t)$, the waveform values are

$$\overline{x} = \frac{X}{\pi/2} \approx 0.637 \cdot X \;;\; \tilde{x} = \frac{X}{\sqrt{2}} \approx 0.707 \cdot X$$

The performance parameters to be minimized in design are form factors, κ and specifically in inverters, crest factors, χ;

$$\kappa = \frac{\tilde{x}}{\overline{x}} = \frac{\pi}{2 \cdot \sqrt{2}} \approx 1.111 \;;\; \chi = \frac{\hat{x}}{\tilde{x}} = \sqrt{2} \approx 1.414 \Rightarrow \chi \cdot \kappa = \frac{\hat{x}}{\overline{x}} = \frac{\pi}{2} \approx 1.571$$

For power, the above parameters are squared. For sine-wave current and voltage, power components must be able to sustain twice ($\chi^2 = 2$) the power during peaks than the average. At 50 or 60 Hz, the peak regions of the waveform last longer than the usual thermal time-scale of power circuits, and components must be rated in power for the peak. A doubled component power rating relative to the inverter rating significantly increases cost and size, and is why low-cost inverter designs settle for square waves.

A waveshape with both sine-wave and square-wave properties overcomes some of the square-wave disadvantages while reducing the figures of demerit and noise. To reduce χ^2 and limit higher harmonics, a third harmonic is added to the fundamental sine wave, shown in Fig. 1.

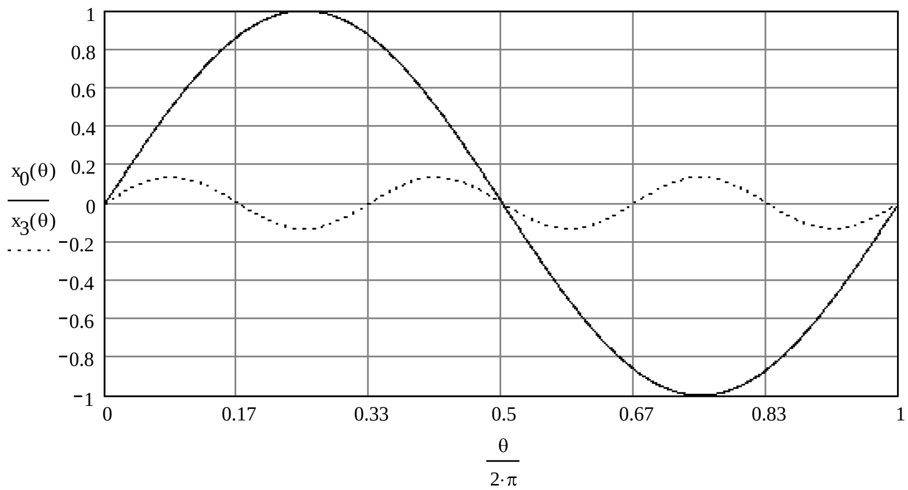

Fig. 1. A sine wave of fundamental (×1) frequency, x_0 and one of ×3 frequency, x_3, the third harmonic of x_0. The third harmonic is in-phase with the fundamental so that it subtracts from the sine-wave peaks. The combined waveform is shown in Fig. 2 for the positive half-cycle.

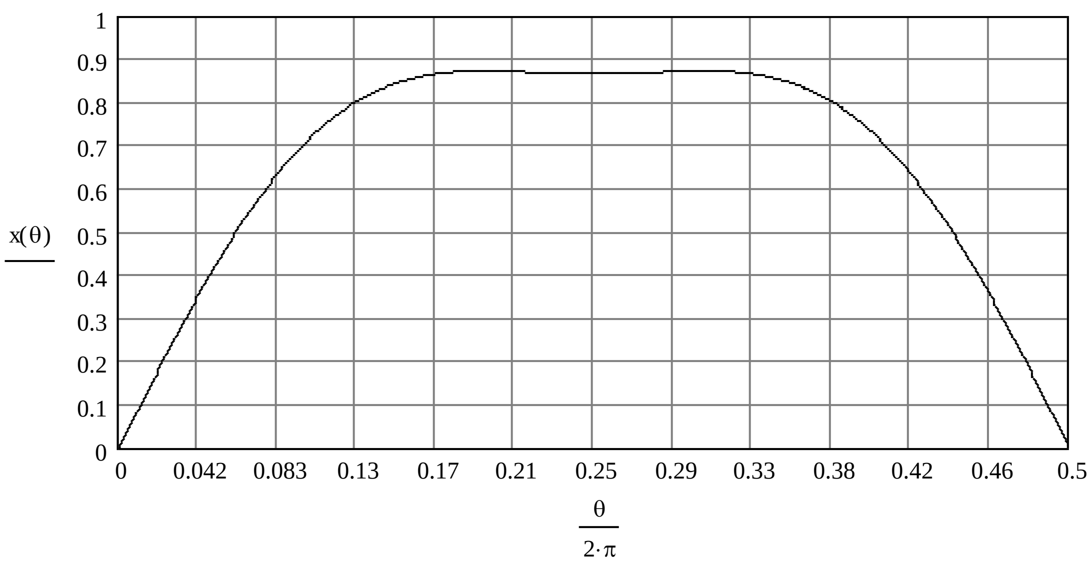

Fig. 2. Third-harmonic sine wave (3HSW), positive half-cycle. Adding fundamental and third harmonic in the right proportions results in an approximate square wave with harmonics limited to the third harmonic.

This *third-harmonic sine wave* (3HSW) has a slightly indented top which is otherwise nearly flat over the middle half of the half-cycle. The new waveform is described by

$$x(t) = X \cdot (\sin(\omega \cdot t) + a \cdot \sin(3 \cdot \omega \cdot t))$$

By applying the trigonometric relationship,

$$\sin(3\cdot\omega\cdot t) = 3\cdot\sin(\omega\cdot t) - 4\cdot\sin^3(\omega\cdot t)$$

$x(t)$ can be rewritten as

$$x = 4\cdot X\cdot\sin(\omega\cdot t)\cdot(1 - a\cdot\sin^2(\omega\cdot t))$$

In this equation a is chosen to satisfy the constraint that the amplitude of the third harmonic equals the difference in value of the fundamental between where the third-harmonic crosses zero (at 60°) and the peak. At 60°, the fundamental sine has a value of sin(60°) = $\sqrt{3}/2 \approx 0.866$. Let the sine amplitude or peak, $X = 1$, and the difference is a;

$$a = 1 - \sin(\pi/3) = 1 - \sqrt{3}/2 \approx 1 - 0.866 \approx 0.134$$

Then substituting a back into the first equation above for x(t), the 3HSW function becomes

$$x = X\cdot\left(\sin(\omega\cdot t) + \left(1 - \frac{\sqrt{3}}{2}\right)\cdot\sin(3\cdot\omega\cdot t)\right)$$

The 3HSW function has two peaks and three zero derivatives per half-cycle. The waveform peaks are of equal value and occur at

$$\theta = \cos^{-1}\left(\pm\sqrt{\frac{9\cdot a - 1}{12\cdot a}}\right) = \cos^{-1}\left(\pm\frac{\sqrt{15 - 6\cdot\sqrt{3}}}{6}\right) \approx \cos^{-1}(\pm 0.358)$$

or approximately at 1.205 rad (69.04°) and 1.937 rad (111.0°). The slight dip at 90° has the value of the fundamental sine wave at 60°, or $X\cdot\sqrt{3}/2 \approx 0.866\cdot X$, and the peaks are at about $0.873\cdot X$.

What are the waveform performance parameters of this new waveform? For the average,

$$\bar{x} = \frac{X}{\pi}\cdot\int_0^{\pi} x(t)\cdot dt = \frac{2}{\pi}\cdot\left(1 + \frac{a}{3}\right)\cdot X = \frac{8 - \sqrt{3}}{3\cdot\pi}\cdot X \approx 0.665\cdot X$$

The RMS value is

$$\tilde{x} = X\cdot\sqrt{\frac{1}{\pi}\cdot\int_0^{\pi}[x(t)]^2\cdot dt} = X\cdot\sqrt{\frac{1}{\pi}\cdot\int_0^{\pi}(\sin^2\theta + 2\cdot a\cdot\sin\theta\cdot\sin 3\cdot\theta + a^2\cdot\sin^2 3\cdot\theta)\cdot d\theta}$$

$$= X\cdot\sqrt{\frac{1}{\pi}\cdot\left[\left(\frac{\theta}{2} - \frac{\sin 2\cdot\theta}{2}\right) + 2\cdot a\cdot\left(\frac{\sin 2\cdot\theta}{4} - \frac{\sin 4\cdot\theta}{8}\right) + a^2\cdot\left(\frac{\theta}{2} - \frac{\sin 6\cdot\theta}{12}\right)\right]_0^{\pi}} = X\cdot\sqrt{\frac{1 + a^2}{2}} \approx 0.713\cdot X$$

The performance parameters are

$$\kappa = \frac{\tilde{x}}{\bar{x}} \approx 1.073\ ;\ \chi = \frac{\hat{x}}{\tilde{x}} \approx \frac{0.873}{0.713} \approx 1.224\ ;\ \chi\cdot\kappa = \frac{\hat{x}}{\bar{x}} \approx \frac{0.873}{0.665} \approx 1.313$$

Compared to a sine wave, 3HSW has a comparable form factor (about 3.4% lower) and average (about 4.6% higher), but its pk/avg ratio, $\chi\cdot\kappa$ is a significant 16.4% lower. The ratio for power is

$$\frac{\hat{P}}{\overline{P}} = \frac{\hat{x}^2}{\tilde{x}^2} = \left(\frac{\hat{x}}{\tilde{x}}\right)^2 = \chi^2$$

The 3HSW crest factor squared, χ^2 is close to 1.5, and is 25% less than for a sine wave.

Table 1 contains the parameters for four waveforms, including bipolar square waves with amplitude *X* and *D* = ½ and at reduced amplitude with larger *D*, as typically found in low-cost inverters. An interesting property of square waves is that

$$\chi = \frac{\hat{x}}{\tilde{x}} = \kappa = \frac{\tilde{x}}{\overline{x}} = \frac{1}{\sqrt{D}}$$

The product of the two parameters results in

$$\chi \cdot \kappa = \frac{\hat{x}}{\overline{x}} = \frac{1}{D} = \left(\frac{\hat{x}}{\tilde{x}}\right)^2, \ \overline{x} = D \cdot \hat{x}$$

Table 1. Inverter waveforms and their performance parameters.

Waveform	$\hat{x}/X$	$\overline{x}/X$	$\tilde{x}/X$	$\kappa = \tilde{x}/\overline{x}$	$\chi = \hat{x}/\tilde{x}$	$\chi \cdot \kappa = \hat{x}/\overline{x}$	χ^2
Sine wave	1	2/π ≈ 0.637	$\sqrt{2}/2$ ≈ 0.707	1.111	$\sqrt{2}$ ≈ 1.414	π/2 ≈ 1.571	2
3HSW	0.873	0.665	0.713	1.073	1.224	1.313	1.498
Normalized 3HSW *X′* = 0.992·*X*	0.866	0.660	$\sqrt{2}/2$ ≈ 0.707	1.073	1.224	1.313	1.498
bipolar square wave *D* = 0.5	1	0.5	$\sqrt{2}/2$ ≈ 0.707	$\sqrt{2}$ ≈ 1.414	$\sqrt{2}$ ≈ 1.414	2	2
bipolar square wave *D* = 0.692	0.85	0.588	$\sqrt{2}/2$ ≈ 0.707	1.202	1.202	1.445	1.445
bipolar square-wave *D* = 0.95	0.726	0.690	$\sqrt{2}/2$ ≈ 0.707	1.026	1.026	1.053	1.053

Table 1 shows that a max-*D* square wave (*D* = 0.95) is optimum from an efficiency and cost standpoint (min χ^2), though (per the result in column 1) it fails to meet amplitude requirements at over 27% below nominal line voltage. (If the loads are all universal-input power converters, then this is the optimum alternative.) The 3HSW is normalized to X′ from X so that its RMS value is equal to the others as the basis for comparison.

The square wave with *D* = 0.5 has the same χ^2 as the sine wave and greatest circuit loss with κ highest of all the waveforms. The only advantage of square waves is through amplitude reduction and increased *D*. The low-cost inverter square wave (*D* ≈ 0.7) reduces the peak value by 18% from that of a sine wave, and with a κ value increase of 8.2%. Consequently, for the same average current, this square wave as a current waveform would dissipate $(1.202/1.111)^2$ or 17.1% more power than a sine wave. This will affect inverter-stage design where this extra loss occurs.

The RMS-normalized 3HSW has a slightly higher average than the sine of 3.8%, giving it a slightly better κ value by 3.4%; 3HSW is comparable in both average and κ. Where the 3HSW is significantly better is in reducing the peak power from that of a sine-wave χ^2 = 2 to 1.5, a significant 25% decrease. Consequently, this waveform is attractive to generate as an alternative to either sine waves or square waves. It shares the EMI advantage of the sine wave in that the 150- or 180-Hz third-harmonic component will produce negligible additional noise for most applications but significantly reduces power-component ratings from what is required for sine waves.

H-Bridge PWM Switching

Having evaluated the waveshapes that we could potentially produce with our inverter H-bridge, we now consider the possible switching techniques we could use to generate these waveforms including the desired 3HSW. The H-bridge, as shown in Fig. 3, has bipolar drive to the output load (shown as a transformer) with one (unipolar) source, V_c = 155 V connected to the top node of the H-bridge. The bottom node is connected to ground. For positive drive, switches Q1 and Q4 are driven on, and for negative drive, switches Q2 and Q3.

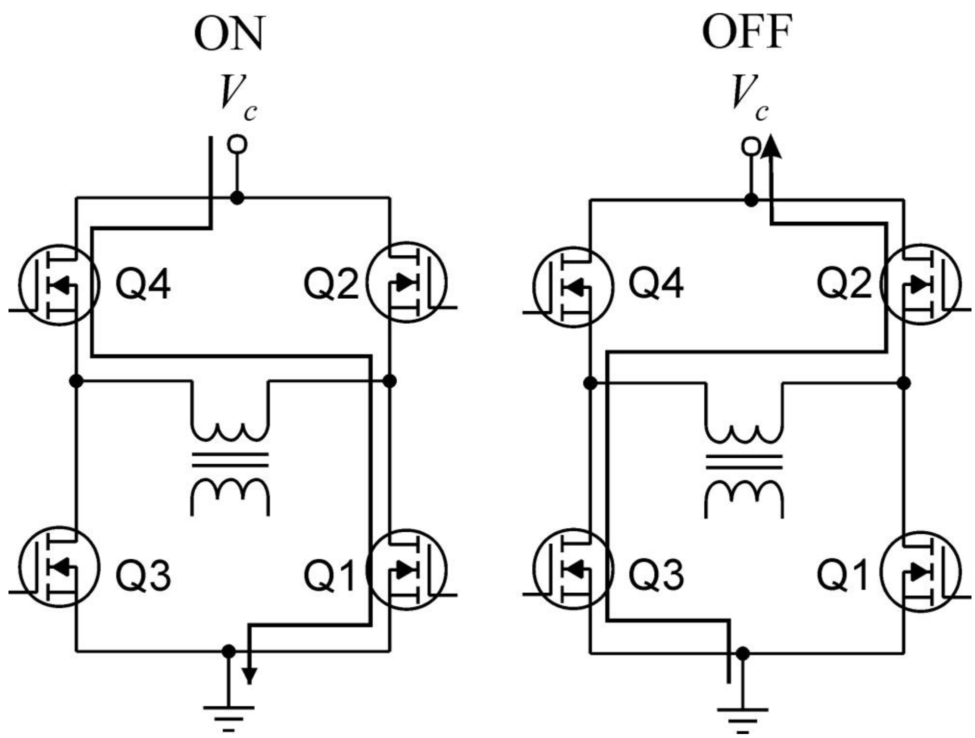

Fig. 3. H-bridge for on-time (ON) and off-time (OFF) for each PWM cycle. During off-time, the body-drain diodes of the MOSFETs conduct to maintain inductive current of the (transformer) load.

Waveforms are generated by PWMing the H-bridge. The PWM switching cycle (not the 60-Hz output cycle) produces an average output voltage of

$$\bar{v}_o = D \cdot V_c$$

One H-bridge switching scheme is two-quadrant (2Q) (sign-magnitude). For circuit design, the two PWM options are high- or low-side PWM. For the sign (polarity) of output and for low-side PWM, select high-side switch Q1 or Q3 to be on. Then PWM the opposing low-side switch Q4 or Q2. During the on-time, sense i_S. During the off-time, output current reroutes through diodes D3 or D1. For high-side PWM, set the polarity (sign) through choice of switch Q1 or Q3, and then PWM (for magnitude) switch Q2 or Q4.

The four-quadrant (4Q) (2's complement) switching scheme PWMs both high- and low-side switches. During PWM on-time, diagonally-opposed switches Q1 and Q4 or Q2 and Q3 are on. During the off-time, Q2 and Q3 or Q1 and Q4 are on. At $D = 0.5$, the switch pairs Q1, Q4 and Q2, Q3 are on equal times, and the average output voltage is zero over the PWM switching cycle, T_s. Of the H-bridge PWM switching schemes, none stands out as clearly the best for all applications. Switching loss is minimized by choosing the scheme with the fewest switching events per cycle, and that is the 2Q scheme.

Two-Parameter PWM

An additional enhancement to PWM comes from the recognition that the PWM waveform has two independent edges within the switching time interval. Ordinarily, the duty ratio, D, is the controlled parameter, while the phase of the on-time pulse is fixed. If the on-time begins the T_s cycle, then the PWM is *trailing edge*, and the off-time follows the on-time.

If instead the trailing edge of the on-time pulse is fixed at the end of T_s, then the T_s interval begins with the off-time and ends when the on-time begins; the PWM is *leading edge*. These are the two extremes within a continuous range of phasing of the on-time pulse within the switching cycle. The fraction of T_s that is the on-time delay, t_d is

$$B = \frac{t_d}{T_s}$$

where t_d is the time from the beginning of the T_s interval to the leading edge of the on-time pulse. An on-time delay of $(D'/2) \cdot T_s$, or $B = D'/2$, is called *centered* PWM.

The additional control that B offers can be used to generate more-precise waveforms. The technique of *selective harmonic elimination* adjusts on- and off-time edges of successive pulses (or equivalently, B and D) to null square-wave harmonics, by making the Fourier-series components of a square-wave cancel each other for the lower harmonics.

This method requires numerical precision and is capable of producing low-distortion sinusoids. What is required to find the pulse widths and spacings is the solution of a system of trigonometric equations, made easier by converting them to Chebyshev polynomials. The resulting math is not unlike that used to design Chebyshev filters.

Two-parameter PWM would most easily be implemented using a fast µC, and would add to the cost of an inverter. This would, however, be a preferred alternative in a laboratory-quality inverter design. (I can't say whether this technique has been implemented yet commercially but it has been discussed in the academic literature as "selective harmonic elimination".)

On the other hand, for a higher-quality replacement of low-cost commercial inverters, a simple multi-step scheme approximating the desired output waveform is synthesized using simple circuitry as will be discussed later. Such a scheme is used in the Volksinverter. The waveform synthesized is the 3HSW.

References

1. "Designing An Open-Source Power Inverter (Part 1): Goals And Specifications" by Dennis Feucht, How2Power Today, May 2021.
2. "Designing An Open-Source Power Inverter (Part 2): Waveshape Selection" by Dennis Feucht, How2Power Today, September 2021.
3. "Designing An Open-Source Power Inverter (Part 3): Power-Transfer Circuit Options" by Dennis Feucht, How2Power Today, April 2022.
4. "Improving Reliability Of Low-Cost Power-Source Inverters" by Dennis Feucht, How2Power Today, October 2020.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For further reading on inverter design, see the How2Power Design Guide, locate the Power Supply Function category and select "DC-AC power inverters."