The Importance Of Knowing Magnetic Core Saturation Field Strength For Accurate Hysteresis Loss Calculation

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Those who are involved in magnetic component design know that it is desirable to have a hysteresis curve available for accurate calculations and for understanding what a magnetic core is capable of. Unfortunately, magnetic core manufacturers very seldom provide full hysteresis loops for their products, limiting the parameters to the saturation flux density, remanence, and permeability.

Despite that limitation, as has been demonstrated in previous work, it is possible to build a hysteresis curve based on these few parameters. Moreover, it is possible to predict the magnetic parameters' behavior at different magnetizing currents. This is very helpful in designing transformers, inductors and electric magnets (including solenoids).

An expression for the hysteresis loop (see reference 1) allows for easy calculation of the hysteresis loop power loss—a crucial characteristic that is necessary to calculate the magnetic core power loss and converter efficiency. Knowledge of the expression for the hysteresis loop shows the core magnetic permeability change with the magnetization field change. The limits of the magnetization field H_{sat} and $-H_{sat}$ directly define the core power loss.

Although the previous work has shown how to calculate core power loss using an expression for the hysteresis loop derived from a core's data sheet specifications, it assumed values of H_{sat} and $-H_{sat}$ based on B_{sat}/μ, without clearly defining how to determine the acceptable value of permeability. In this article, a method is proposed for defining H_{sat} based on an analytic expression for the core permeability dependence on the magnetizing field induced in the core. It assumes dependence of permeability on the applied magnetic field and a value of an *admissible permeability loss* due to the magnetization strength H rise. The $\mu(H)$ function is a hyperbolic one, whose graphical interpretation is not very obvious and thus not easy to perceive.

Using the new expression for H_{sat} derived here, we'll see how it can be applied to the previously derived expression for hysteresis loss in a design example. For the reader's convenience, any formulas from reference 1 required for our calculations are repeated here.

Building A Hysteresis Loop

The referenced article[1] has this set of expressions describing the hysteresis loop:

$$B_1(H_x) = B_{sat}\left[\frac{2}{1+e^{\frac{-(H_x-H_c)}{H_0}}} - 1\right] \tag{1}$$

$$\mathrm{B_2(H_x)} = \mathrm{B_{sat}}\left[\frac{2}{1+e^{\frac{-(\mathrm{H_x+H_c})}{\mathrm{H_0}}}} - 1\right] \tag{2}$$

Here B_{sat} is the core saturation flux density and H_C is the core coercive force. H_0 is a virtual magnetic field strength whose value should be determined by deriving an expression for H_0. H_X is the present magnetic field strength. $B_1(H_X)$ and $B_2(H_X)$ are values for present flux densities for both branches of the hysteresis loop.

The core dynamic permeability μ for each branch is described by derivatives of the above expressions. Please keep in mind that this permeability is a *full* permeability that can be expressed as $\mu = \mu_0 \times \mu_1$ where μ_1 is a relative permeability of the core material in the loop point of interest. When the hysteresis loop branch flattens out, the μ_1 value goes down. Therefore, μ_1 is a function of H_X and $\mu = \mu(H_X)$.

Taking a derivative of each branch (1) and (2) to obtain the instantaneous value of permeability and simplifying results, we get

$$\mu_1(H_x) = \frac{B_{sat}}{2 \cdot H_0 \cdot \cosh\left(\frac{H_c + H_x}{2 \cdot H_0}\right)^2} \tag{3}$$

and

$$\mu_2(H_x) = \frac{B_{sat}}{2 \cdot H_0 \cdot \cosh\left(\frac{H_c - H_x}{2 \cdot H_0}\right)^2} \tag{4}$$

We see that both expressions have a term H_0 that must be found. We can also see that each branch has an offset H_C with respect to the B coordinate. Hence, it is reasonable to find a median graph for $\mu(H_X)$ and use it for finding H_0.

$$\mu_{mean}(H_x) = \frac{\frac{B_{sat}}{2 \cdot H_0 \cdot \cosh\left(\frac{H_c + H_x}{2 \cdot H_0}\right)^2} + \frac{B_{sat}}{2 \cdot H_0 \cdot \cosh\left(\frac{H_c - H_x}{2 \cdot H_0}\right)^2}}{2}$$

Simplifying the above expression, we get

$$\mu_{mean}(H_x) = \frac{B_{sat} \cdot \left(2 \cdot \cosh\left(\frac{H_c + H_x}{2 \cdot H_0}\right)^2 + 2 \cdot \cosh\left(\frac{H_c - H_x}{2 \cdot H_0}\right)^2\right)}{8 \cdot H_0 \cdot \cosh\left(\frac{H_c + H_x}{2 \cdot H_0}\right)^2 \cdot \cosh\left(\frac{H_c - H_x}{2 \cdot H_0}\right)^2} \tag{5}$$

At the beginning of the coordinates where $H_X = 0$ and $H_C = 0$, the initial value of the magnetic core permeability μ_{ini}, which is a datasheet parameter, will be

$$\mu_{ini}(H_x) = \frac{B_{sat} \cdot \left(2 \cdot \cosh\left(\frac{0 + 0}{2 \cdot H_0}\right)^2 + 2 \cdot \cosh\left(\frac{0 - 0}{2 \cdot H_0}\right)^2\right)}{8 \cdot H_0 \cdot \cosh\left(\frac{0 + 0}{2 \cdot H_0}\right)^2 \cdot \cosh\left(\frac{0 - 0}{2 \cdot H_0}\right)^2}$$

Simplifying, we get

$$\mu_{ini} = \frac{B_{sat}}{2 \cdot H_0} \tag{6}$$

Therefore

$$H_0 = \frac{B_{sat}}{2 \cdot \mu_{ini}} \tag{7}$$

Now, knowing the expression for H_0, we can plug it in into equations (1) and (2) and obtain expressions for both branches' flux densities as follows:

$$B_1(H_x) = B_{sat}\left[\frac{2}{1 + e^{\frac{-(H_x - H_c)}{\frac{B_{sat}}{2 \cdot \mu_{ini}}}}} - 1\right] = -B_{sat} \cdot \tanh\left[\frac{\mu_{ini} \cdot (H_c - H_x)}{B_{sat}}\right] \tag{8}$$

$$B_2(H_x) = B_{sat}\left[\frac{2}{1 + e^{\frac{-(H_x + H_c)}{\frac{B_{sat}}{2 \cdot \mu_{ini}}}}} - 1\right] = B_{sat} \cdot \tanh\left[\frac{\mu_{ini} \cdot (H_c + H_x)}{B_{sat}}\right] \tag{9}$$

Accordingly (3) and (4) can then be expressed as

$$\mu_1(H_x) = \frac{B_{sat}}{2\cdot\frac{B_{sat}}{2\cdot\mu_{ini}}\cdot\cosh\left(\frac{H_c+H_x}{2\cdot\frac{B_{sat}}{2\cdot\mu_{ini}}}\right)^2} = \frac{\mu_{ini}}{\cosh\left[\frac{\mu_{ini}\cdot(H_c+H_x)}{B_{sat}}\right]^2} \qquad (10)$$

$$\mu_2(H_x) = \frac{B_{sat}}{2\cdot\frac{B_{sat}}{2\cdot\mu_{ini}}\cdot\cosh\left(\frac{H_c-H_x}{2\cdot\frac{B_{sat}}{2\cdot\mu_{ini}}}\right)^2} = \frac{\mu_{ini}}{\cosh\left[\frac{\mu_{ini}\cdot(H_c-H_x)}{B_{sat}}\right]^2} \qquad (11)$$

And the mean value for the core permeability is

$$\mu_{mean}(H_x) = \frac{\frac{\mu_{ini}}{\cosh\left[\frac{\mu_{ini}\cdot(H_c+H_x)}{B_{sat}}\right]^2}+\frac{\mu_{ini}}{\cosh\left[\frac{\mu_{ini}\cdot(H_c-H_x)}{B_{sat}}\right]^2}}{2} \qquad (12)$$

The plot for equations (10), (11) and (12) is shown in Fig. 1

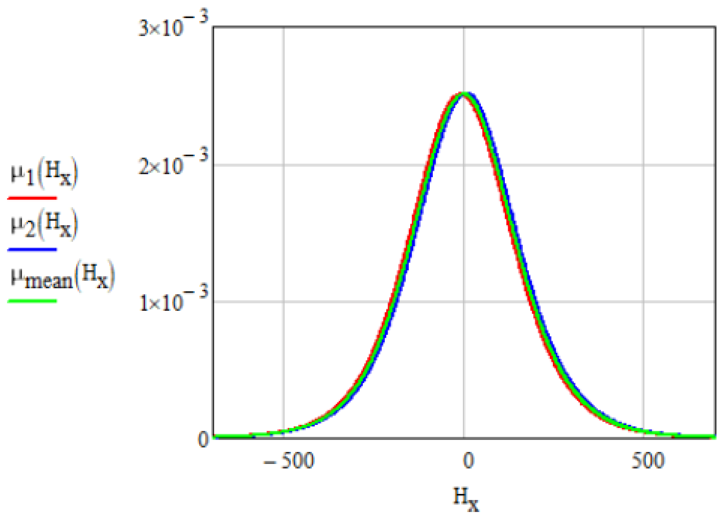

Fig. 1. The mean core permeability depends on the magnetizing field and shows how the core saturates while the magnetic field increases. All values are in SI, so magnetic field strength is in A/m, flux density in T, absolute permeability in H/m and so on.

Now, using Fig. 1 we can assign the saturation value to the mean core permeability—$\mu_{meansat}$—and solve (13) with respect to H_X that will be the magnetizing field value H_{sat} causing the core saturation. Proper selection of the $\mu_{meansat}$ level depends on the designer's skill, experience, and the set-up's affordable power loss. (I elaborate on the selection of $\mu_{meansat}$ for the H_{sat} calculation in the design example presented below.)

$$\mu_{meansat} = \frac{\frac{\mu_{ini}}{\cosh\left[\frac{\mu_{ini}\cdot(H_c+H_x)}{B_{sat}}\right]^2}+\frac{\mu_{ini}}{\cosh\left[\frac{\mu_{ini}\cdot(H_c-H_x)}{B_{sat}}\right]^2}}{2} \qquad (13)$$

Solution of equation (13) for H_X gives eight values, and only two of them are real. For simplicity, we will leave in the real solutions only.

$$H_{sat} := \begin{pmatrix} \dfrac{B_{sat}\cdot \operatorname{acosh}\left(\sqrt{\dfrac{\mu_{meansat}+\mu_{ini}\cdot\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)-\mu_{meansat}\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\dfrac{\sqrt{2}\cdot\sqrt{\mu_{ini}^{2}\cdot\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\mu_{ini}^{2}+2\cdot\mu_{ini}\cdot\mu_{meansat}-2\cdot\mu_{ini}\cdot\mu_{meansat}\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)}}{2}}{2\cdot\mu_{meansat}}}\right)}{\mu_{ini}} \\ -\dfrac{B_{sat}\cdot \operatorname{acosh}\left(\sqrt{\dfrac{\mu_{meansat}+\mu_{ini}\cdot\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)-\mu_{meansat}\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\dfrac{\sqrt{2}\cdot\sqrt{\mu_{ini}^{2}\cdot\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\mu_{ini}^{2}+2\cdot\mu_{ini}\cdot\mu_{meansat}-2\cdot\mu_{ini}\cdot\mu_{meansat}\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)}}{2}}{2\cdot\mu_{meansat}}}\right)}{\mu_{ini}} \end{pmatrix}$$

Design Example—Calculating Magnetic Field Saturation And Core Loss Values

Having derived an expression for H_{sat}, we will now demonstrate the use of this expression in a design example where we first calculate magnetic field saturation value and then use it to calculate magnetic core hysteresis loss.

In this example, we assume the use a Magnetics ferrite R-material 2915 core[2] for which the following parameters are specified.

B_{sat} = 0.47 T

μ_r = 2000

$\mu_{ini} = \mu_r \bullet \mu_0$ where $\mu_0 = 4\pi \times 10^{-7}$ H·m^{-1}

B_r = 0.02 T

H_c = 7.966 A/m

$\mu_{meansat} = 0.707 \bullet \mu_{ini}$ (see note below)

l_{mag} = 73.2 mm

S_{mag} = 74.9 mm^2

F = 50 kHz

P_1 = 600 W

$Vol_{mag} = S_{mag} \bullet l_{mag}$ =5.483 x 10^{-3} L

Note: From the graph of $\mu_{meansat}$ above in Fig. 1 we see the permeability changing from its peak value corresponding to a μ_r maximum value of 2000 down to 1. The change is huge. We have to agree that saturation occurs when e.g. 30% of all domains have changed their spatial orientation and cannot further participate in the magnetization process. This is why I selected further the core permeability decrease down to 0.707 of the initial value.

$$H_{sat} := \begin{pmatrix} \dfrac{B_{sat}\cdot \operatorname{acosh}\left(\sqrt{\dfrac{\mu_{meansat}+\mu_{ini}\cdot\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)-\mu_{meansat}\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\dfrac{\sqrt{2}\cdot\sqrt{\mu_{ini}^{2}\cdot\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\mu_{ini}^{2}+2\cdot\mu_{ini}\cdot\mu_{meansat}-2\cdot\mu_{ini}\cdot\mu_{meansat}\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)}}{2}}{2\cdot\mu_{meansat}}}\right)}{\mu_{ini}} \\ -\dfrac{B_{sat}\cdot \operatorname{acosh}\left(\sqrt{\dfrac{\mu_{meansat}+\mu_{ini}\cdot\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)-\mu_{meansat}\cosh\left(\dfrac{2\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\dfrac{\sqrt{2}\cdot\sqrt{\mu_{ini}^{2}\cdot\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)+\mu_{ini}^{2}+2\cdot\mu_{ini}\cdot\mu_{meansat}-2\cdot\mu_{ini}\cdot\mu_{meansat}\cosh\left(\dfrac{4\cdot H_c\cdot\mu_{ini}}{B_{sat}}\right)}}{2}}{2\cdot\mu_{meansat}}}\right)}{\mu_{ini}} \end{pmatrix}$$

Hence

$$H_{sat} = H_{sat_0} = 113.285\,\frac{A}{m}$$

where H_{sat_0} means the solution for the upper equation.

There are two real solutions symmetrical with respect to the B coordinate. Using equations (8) and (9),

$$B_1(H_x) = -B_{sat}\cdot\tanh\left[\frac{\mu_{ini}\cdot(H_c-H_x)}{B_{sat}}\right]$$

$$B_2(H_x) = B_{sat}\cdot\tanh\left[\frac{\mu_{ini}\cdot(H_c+H_x)}{B_{sat}}\right]$$

we can plot the hysteresis loop with the results shown in Fig. 2.

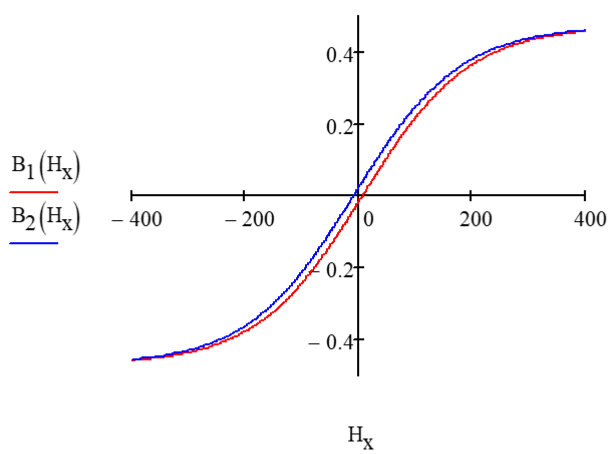

Fig. 2. Hysteresis loop of the magnetic core in the example.

Having expressions for $B_1(H_X)$ and $B_2(H_X)$, we can now use the following formula from reference 1 to determine hysteresis loss, which is loop power loss:

$$P_{hyst\_loss} = \int_{-H_{sat}}^{H_{sat}} f_{SW}\cdot Vol_{mag}\cdot\left(B_2(H_X)-B_1(H_X)\right)dH_X$$

From reference 1, we also know that the magnetic core volume necessary for transferring power P_1 can be defined as

$$\mathrm{Vol_{mag}} = \frac{\mathrm{P_1} \cdot \mu_0 \cdot \mu_r}{4 \cdot \mathrm{B_{sat}}^2 \cdot \mathrm{f_{SW}}}$$

From the conditions defined above, we have f_{SW} = 50 kHz, P_1 = 600 W and the other parameters for calculating Vol_{mag}. So in our particular example, the minimum magnetic core volume necessary for non-saturable operation is

$$\mathrm{Vol_{mag}} = \frac{\mathrm{P_1} \cdot \mu_0 \cdot \mu_r}{4 \cdot \mathrm{B_{sat}}^2 \cdot \mathrm{f_{SW}}} = 0.034\ \mathrm{L}$$

However, the actual magnetic core volume based on the core geometry is

$Vol_{mag\_actual} = l_{mag} \bullet S_{mag} = 5.483 \times 10^{-3}$ L = 0.005483 L

So for the given power level, the volume of the selected core is too small and we will have to use a magnetic core having volume Vol_{mag} to ensure its non-saturable operation.

Assuming the Vol_{mag} value of 0.034 L, hysteresis loss can be defined this way:

$$\mathrm{P_{hyst\_loss}} = \int_{-\mathrm{H_{sat}}}^{\mathrm{H_{sat}}} \mathrm{f_{SW}} \cdot \mathrm{Vol_{mag}} \cdot \left(\mathrm{B_2(H_X)} - \mathrm{B_1(H_X)}\right) \mathrm{dH_X} = 13.825\ \mathrm{W}$$

Efficiency

$$\eta = \frac{\mathrm{P_1}}{\mathrm{P_1} + \mathrm{P_{hyst\_loss}}} = 0.977$$

It is very important to know not only the magnetic core parameters, but also the magnetizing field strength to properly estimate the hysteresis loss.

References

1. "Analysis Of Core Hysteresis Loss Underscores Transformer Efficiency Challenges" by Gregory Mirsky, How2Power Today, February 2023.
2. Ferrite core R Material page and 0R42915TC core datasheet, Magnetics website.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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