

Mythology In Power Magnetics

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Magnetic components appear to be so simple—just two parts, a core and some wire wrapped around it. How could that be very complicated? If you ask this question of yourself seriously enough, you begin your own descent into the abyss of magnetics design. As a “recovering magnetaholic,” I have learned that magnetics really is simple, but the path to simplicity is fraught with misleading ideas. Some of these ideas are partially true, but misleading in the ways they are usually expressed; while others are not true at all, though they may be widespread. More importantly, some basic concepts that should be widely known are not. This article is a chat about some of them.

Magnetics Science Vs. Magnetics Optimization

A few years ago, I began design work on a battery converter for an off-grid inverter. The converter has low-input-resistance (low V , high I), from a 24-V battery and at 1 kW, by Watt’s Law, takes in about 42 A. After some research, I concluded that the best power circuit is that of a *boost push-pull* (BPP), which is a common-active PWM-switch converter with a transformer, as shown in Fig. 1.

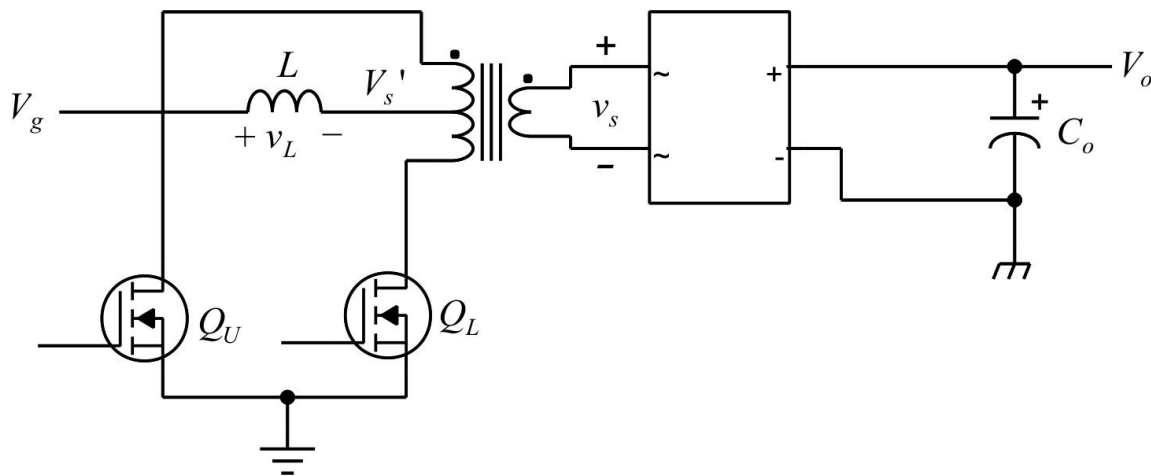


Fig.1. Boost push-pull (BPP) converter, the “optimal” choice for low-input-voltage, high-current conversion?

The converter has input inductor L and a center-tapped transformer, giving it essentially two primary windings and a 170-V secondary winding. The secondary voltage referred to the primary appears at the center-tap as $V_s' = n \cdot V_s$ where turns ratio $n = N_p/N_s = 1/5$ —a “step-up” transformer. Then $V_s' = 34$ V and is above the maximum specified input voltage V_g of 30 V.

The amount of power that the inductor must be capable of transferring decreases as V_s' decreases, which reduces the inductor voltage v_L and flux change $\Delta\lambda$ and thus its transfer power. However, unless $V_s' > V_g$, the transfer circuit quits boosting and the direction of power flow reverses. With changes in n , inductor power is greatly affected but not so much for transformer power because all power transferred between input and output ports must go through it.

The circuit poses questions about the design of the inductor and transformer and how to optimize them. These seemingly reasonable questions have not been given convincing answers. Textbooks do not give them because they do not address design *optimization* except in a somewhat facile sense. They concentrate primarily on the science of magnetics in its application to engineering. Engineering, however, is not only about science but is distinctively about design, optimization, and generally, about achieving goals that solve human physical problems.

Furthermore, at power-on and power-off and during load faults (output shorted), $V_s' < V_g$ and inductor current ratchets upward, out of control. The basic push-pull boost circuit can have excessive current under these conditions because ratcheting is caused by driver loop delay; it takes a loop delay time to shut off the power switches, and during this time, inductor (or transformer magnetizing) current can increase more than it decreases in a switching cycle thereby causing the current ratcheting effect. Some way of avoiding it is required.

The solution that was found to be optimal (in the Volksinverter series of articles just completed ^[1]) was a Weinberg circuit, where the inductor is given a secondary winding that delivers current through a diode either to the output or back to the input port. Instead of operating with non-overlapping (CP or buck) push-pull switch drive, it has dual operation that can transition continuously to a CA or boost converter with overlapping switch conduction for $V_s' > V_g$. Thus, both input and output port ranges of voltages and currents must be considered in a converter design. Sometimes the exceptional yet critical states of a circuit are overlooked.

Maximum-Power Mythology

One of the semi-myths of magnetics design that I had carried in my notebook and mind for many years is the belief that maximum power is transferred through a transformer when the winding and core losses are equal, or average $P_w = P_c$. This is demonstrated by using the maximum power-transfer theorem (which is better called the maximum output power theorem), by letting winding loss be included in a series winding resistance with a shunt resistance for core loss, all referred to the secondary winding, as shown in Fig. 2.

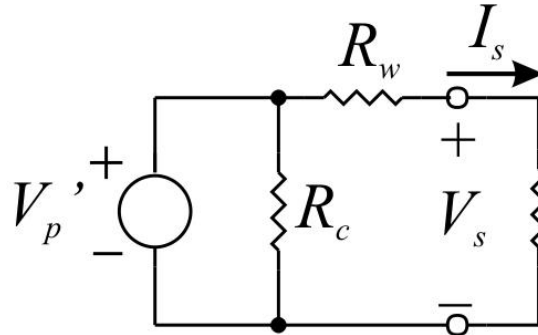


Fig. 2. Interwinding power transfer model, referred to the secondary winding with a resistive load, secondary (load) current of I_s and voltage of V_s . R_c is the secondary-referred core resistance representing core power loss, and R_w is the secondary-referred winding resistance of both windings combined.

Core-loss resistance R_c is across secondary-referred primary winding voltage V_p' . Because core loss varies directly with flux change, which is proportional to voltage, then the loss can be modeled in R_c . The dictum of maximum power transfer under the condition $P_w = P_c$ is true only at zero power, or 100% efficiency. Otherwise, maximum power is transferred—that is, maximum efficiency occurs elsewhere. In the example given here, $P_w < P_c$ at maximum efficiency, as seen in Fig. 3, though for typically much higher efficiency, $P_w \approx P_c$.

In reference [4], the optimal power-loss ratio ψ of winding-to-core loss varies around one, depending on the extent of voltage drop across winding resistance and the winding-to-core resistance ratio β . For the general model (with individual primary and secondary winding resistances) referred to the primary side, $\psi_{\max} = \beta + 1 > 1$, which makes $P_w > P_c$ but not by much.

The primary winding is driven by a constant voltage source, referred to the secondary as V_p' with a secondary-winding terminal voltage of V_s . The secondary output power is

$$P_s = V_s \cdot I_s, \quad I_s = \frac{V_p' - V_s}{R_w}$$

As R_w decreases, P_s increases along with V_s and I_s . P_c is a function of V_p' and P_w is a function of both V_p' and V_s . Core power loss P_c and winding power loss P_w are

$$P_c = \frac{V_p'^2}{R_c}; \quad P_w = \frac{(V_p' - V_s)^2}{R_w}$$

The total transformer power loss is

$$P_t = P_c + P_w = \frac{V_p'^2}{R_c} + \frac{(V_p' - V_s)^2}{R_w} = \frac{V_p'^2}{R_c} + I_s^2 \cdot R_w$$

The primary-winding input power is

$$P_p = P_s + P_t$$

Power transfer as an output/input ratio is the same as *efficiency*,

$$\eta = \frac{P_s}{P_p} = \frac{P_s}{P_s + P_t} = \frac{V_s \cdot I_s}{V_s \cdot I_s + P_c + I_s^2 \cdot R_w}$$

Plots for the circuit are shown in Fig. 3 for constant $V_p' = 10$ V, $R_c = 9$ Ω , and $R_w = 1$ Ω . Power quantities are in watts and η is in percentage. P_c and P_w cross at 3.33 A while η peaks at 2.4 A with $\eta = 51.9\%$. Where $P_c = P_w$, $\eta < \eta_{\max}$ by about 1.9%.

The cause of the error is that the maximum power-transfer theorem from introductory passive-circuits courses is based on a different circuit. The theorem is derived from a voltage source having a series resistance connected to a resistive load. The difference is that shunt core resistance R_c is not included in the theorem. If core loss is negligible for high- η design, it is approximately correct. However, to believe that it has been derived as exact is “mythological”.

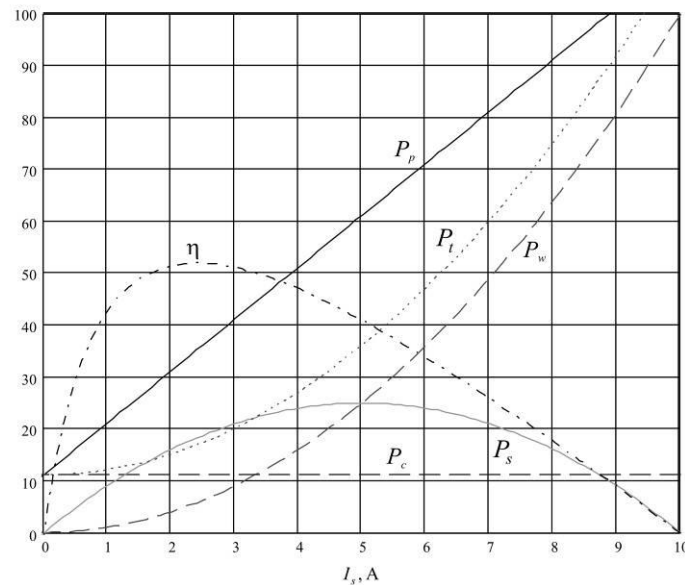


Fig. 3. Graph showing for the example given that maximum power is not transferred when core and winding losses are equal. P_t intersects P_s at two values of I_s , neither of which is at peak efficiency, η . At one intersection, $P_c = P_w$.

There is also a subtle calculus error in the commonplace derivation of maximum η .^[2] Hence, one longstanding interwinding power-transfer proposition found in much of the power-electronics literature is not really true. For high efficiency, it is approximately true which is why it has been possible to promulgate it all these years without undue suspicion. Yet if you suppose it is *always* true when analyzing your converter design over its full input-current range, you will find that the efficiency falls off rather abruptly at low current. Here the old dictum begins to fail noticeably.

Optimal Waveshape For A Core Material

Another underappreciated loose end is the relationship between circuit waveforms and optimal core material. Seemingly, there should be no direct relationship, but there is.^[5] The question of how to select the right core material for a given magnetics design starts with the basic limitations on cores.

Frequencies much above audio eliminate 60-Hz transformer cores such as 3% Si steel. All of the low-frequency materials have excessive hysteresis loss for operation at power-converter switching frequencies. Operation at the highest possible frequency maximizes power-transfer density because power transfer in a converter is

directly proportional to it. Frequency is also related to the more basic core limitation of power loss and maximum allowable core temperature.

The other basic core limitation is saturation, in which its magnetic properties diminish. Power loss limits a combination of the frequency and $\hat{B}_{\sim} = \Delta B/2$, the ripple amplitude of the core field density. Saturation limits static field intensity \bar{H} , and the two combined limit *ripple factor* γ . In field quantities,

$$\gamma = \frac{\hat{i}_{\sim}}{\bar{i}} = \frac{\hat{\phi}_{\sim} / \mathcal{L}}{\bar{H} \cdot l} = \frac{\hat{B}_{\sim} \cdot A / \mathcal{L}}{(\bar{B} / \mu) \cdot l} = \frac{\hat{B}_{\sim}}{\bar{B}}$$

where $\mathcal{L} = \mu \cdot A / l$ = field inductance, A = magnetic cross-sectional area, and l = magnetic path length.

The ripple factor of the winding current waveform is determined by core loss in the numerator and saturation in the denominator. Substitute the maximum allowable values for a given core material, and γ_{opt} for that particular material results. This γ is optimum because it allows maximum energy transfer through the core: maximum ΔB at maximum H for which

$$\Delta W = \Delta B \cdot \bar{H}$$

Ripple factor is a significant core material performance parameter which, to my knowledge, has not been made explicit in the development of power magnetics. Sanjaya Maniktala noted it in passing in his books,^[3] but general awareness in the field has yet to take place.

Winding Area Allotment

Another loose end that relates to both converter circuit and transformer design is how much area should optimally be allotted to each winding. Usually, there is nothing mysterious about this; the primary and secondary windings should each be allotted equal area.

The rationale is simple. Power is being transferred from primary to secondary winding and except for the loss of a negligible amount of power as heat in the transfer, they are equal. Transformer thermal design is optimized by having no part of the transformer hotter than any other part. Although this is an ideal, it is approached by designing for equal power-loss density.

To achieve this for windings of near-equal power, their areas should also be made equal. In a sequential winding configuration, the primary is wound first for highest power transfer to the core, and it is thermally the farthest from ambient. For this kind of transformer design, the primary winding might be given somewhat larger area to reduce its power-loss density and hence its temperature.

However, the heat path of the primary is almost completely through the metal of the secondary winding. This heats the secondary with both its own loss and that of the primary winding, and argues for a lower power density and more area for it. Overall, allotting equal areas for primary and secondary windings is roughly optimal.

A thunderclap is now heard in this otherwise placid scenario when the boost push-pull (BPP) power-transfer circuit is considered. The complication is that it has (functionally) two identical primary windings and one secondary winding. The primary windings alternate in conduction cycles while the secondary conducts each cycle. The primary-side switch current waveforms have three levels, not the familiar two levels of square-waves.

The fractional allotment of total core window area A_w of a winding is k_{ww} : $A_{wp} = k_{wp} \cdot A_w$; $A_{ws} = k_{ws} \cdot A_w$. Then the area ratio of primary to secondary windings, which in most transformers is optimally 1, is

$$Y = \frac{A_{wp}}{A_{ws}} = \frac{2 \cdot \tilde{i}_p}{\tilde{i}_s / n} = n \cdot \sqrt{D'(1+D')} \cdot \frac{V_s}{V_g} = \sqrt{D'(1+D')} \cdot \frac{V_s'}{V_g}, \quad V_s = 170 \text{ V}; 1/n = 5; V_s' = n \cdot V_s = 34.0 \text{ V}$$

(In the Volksinverter series of articles, the waveform equations are derived, but here they are given.) The primary and secondary currents are \tilde{i}_p and \tilde{i}_s , converter PWM duty-ratio = D , $D' = 1 - D$, n = transformer turns ratio of either primary winding to the secondary winding, V_s = secondary voltage amplitude, V_g = converter input voltage, and V_s' is the secondary voltage, referred to the primary winding and probed at the center-tap of the two primary windings. Some area math establishes the basic area relationships:

$$A_w = A_{wp} + A_{ws} \Rightarrow A_{wp} = A_w - A_{ws}; \quad \frac{A_{ws}}{A_w} = \frac{1}{Y+1}; \quad \frac{A_{wp}}{A_w} = \frac{Y}{Y+1}$$

The Voltsinverter-specified input voltage range is 20 V to 30 V. Over this range, the optimal Y changes, but as a transformer, n and Y and the other parameters are fixed in design. Our challenge is picking the best compromise optimum from the table of values.

Table. Winding area allotments for BPP.

$V_g, \text{ V}$	D'	Y	$\frac{A_{wws}}{A_w}$	$\frac{A_{wvp}}{A_w}$
20	0.588	1.643	0.378	0.622
25	0.735	1.536	0.394	0.606
30	0.882	1.460	0.4065	0.5935

The choice is to allot $(0.6) \cdot A_w$ to the primary windings, with $(0.3) \cdot A_w$ to each, and $(0.4) \cdot A_w$ to the secondary winding, as shown in Fig. 4

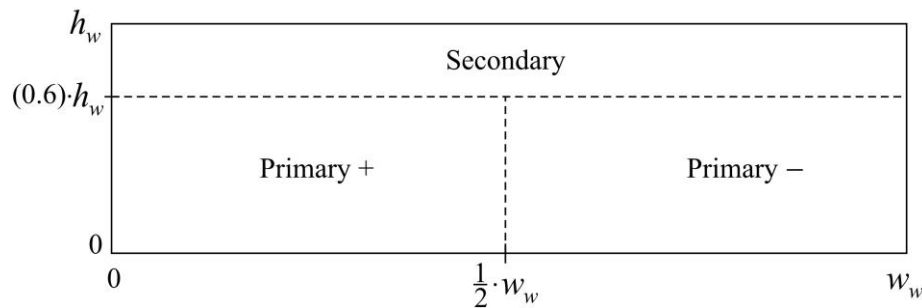


Fig. 4. Boost push-pull converter transformer primaries and secondary winding-area allotment.
Primary windings have $k_{wvp} = 0.3$ and secondary winding has $k_{wvs} = 0.4$.

Departure from $Y = 1$ is caused by the difference in primary and secondary waveforms; they are not the same in waveshape, and this affects the optimal Y . Usually in switching transfer circuits, the primary and secondary waveshapes are close to square-waves (with some slope from magnetizing-current ripple), but whenever a power-transfer circuit has different waveshapes across windings, the equal-area optimization might not be valid.

Optimal Core Shapes

Charles Sullivan at Dartmouth College and his student R. Jensen did a computer-simulation study of core shapes that minimize thermal resistance. A geometric optimization of core shape involves the winding window aspect ratio of width (the dimension along which winding turns proceed) to height, which affects the number of layers, is optimally between 1 and 2 for minimum P_w when dynamic resistance is taken into account.^[5]

Commercial cores are in the range of 2 to 5 and typically around 3, including EE, ETD (4 to 5), EC, RM (2 to 3) and PQ (1.5 to 3) shapes, resulting in about a third greater loss. Although a large width/height aspect ratio reduces the number of winding layers and eddy-current resistance ratio F_R , it also increases the winding length for a given window relative to the window area, thus increasing winding resistance and winding loss.

RM and PQ shapes are found in power inverters such as the Statpower 500 and its equivalent Samlex PS-24175A, but the core shapes this study found to be less thermally efficient, such as ETD and EFD, are also popular and not relegated to obsolescence. Perhaps that will change as these findings diffuse into the broader power-electronics design world and more research clarifies the power-loss picture as it relates to core shapes.

How To Avoid Mythology

Acquiring misleading ideas about magnetics can be avoided with a solid grounding in the fundamental principles that will be as true a century from now as they were a century ago (had they only been known). In surveying the literature, there are two textbooks and a research site that I found to be the most helpful, all from leading magnetics researchers and IEEE Fellows.

The most rigorous and complete book I found on basic magnetics is by Marian K. Kazimierczuk of Wright State U. titled *High-Frequency Magnetic Components*, Second Edition, Wiley, 2014. He has worked out all the equations relating to the fundamentals of component *analysis*, and some for design.

The second book is thinner and has a different emphasis to it, by W. Gerard Hurley in Galway, Ireland, and Werner Wölflé, *Transformers and Inductors for Power Electronics: Theory, Design and Applications*. It has slightly less math density and covers some different topics than Kazimierczuk's book.

And finally, from Charles Sullivan's website in the Thayer School of Engineering at Dartmouth C. you can download his main research papers on magnetics.^[7] Sullivan is primarily interested in how to minimize core and winding losses by reducing the eddy-current effects through wire strand twisting and braiding, and also in how core dimensions affect core loss. These three sources provide a foundation for proceeding to design optimization. More detail on that is in my book, *Power Magnetics Design Optimization*.^[6] If you want a PDF copy (the paper version sustains a printing cost), contact me through the inquiry link^[8] on the Innovatia website.

References

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7. <http://thayer.dartmouth.edu>
8. Contact the author [here](#).

About The Author



Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For more on magnetics design, see these How2Power Design Guide search [results](#).