Optimized Magnetics Winding Design (Part 1): A Discovery Over Fifty Years Late

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An idea whose time has come, such as calculus in the days of Newton and Leibnitz, can sometimes be slow in its diffusion. One simple idea involving the design of magnetic components was delayed in our time by over a half century.

In the mid-1960s, P.J. Dowell published a landmark paper through the IEE in Britain about how to calculate the increase in resistance of conductors caused by eddy currents. A current of varying amplitude in a wire can induce by a magnetic field B into itself, as dB/dt, eddy-current loops that oppose the central current and cause it to crowd to the outside of the wire.

With a lower current density in the center, the effective cross-sectional area of the wire is reduced—hence its increase in resistance, approximated in a wire as that of an area one *skin depth* deep from the wire surface. The resistance of the cross-sectional skin-depth area is that of the eddy-current resistance.

Dowell derived a 1D field solution for a pair of parallel plates conducting current, much like the layers of windings in a transformer. Magnetics textbooks routinely derive and explain it, and how it can be applied to magnetic component *analysis*, which is to say the calculation of winding resistance and therefore power losses for a given transformer design. Yet there is also an obvious way to apply it for magnetics design *optimization*, which minimizes winding resistance while applying constraints on certain winding parameters.

The concepts presented in this new series may be familiar to those who have read my previous works on winding design such as references 1 and 2. However, the material is presented here in more of a tutorial format with further explanation of key points.

This part 1 begins with an explanation of how to apply Dowell's equations to determine optimal wire size at a given frequency, and then extends this approach for use with wire bundling and for a specified winding window area.

Using Dowell's Curves to Find Optimal Wire Size At A Constant Frequency

Most circuit designers steer clear of magnetic component design, except in much-simplified ways. I vividly recall in my younger teens thinking about how transformers were the least interesting of all electronics—merely a core of metal wrapped with some wire. Transformers lacked the complexity and intrigue of computers and color televisions.

In ordinary circuit design, wire has essentially zero resistance, and to contemplate its actual resistance is to become concerned with trifles, so it seemed. But in more recent years, involvement in power electronics led me to wonder how best to design magnetic components—transformers and inductors—and this led to an "obvious" discovery.

The most basic limitation on magnetic parts is the temperature where either magnetic properties or structures themselves fail. Transformers in power converters are basically power-transfer devices. Their temperature rises from heat caused by power dissipation in both windings, the electrical part of the transformer, and core, the magnetic part. Dowell's equation gives the resistance factor by which eddy currents increase wire resistance. At a fixed switching frequency (which is typical of most converters) it is the unitless function F_r. Thus, winding resistance,

$$R_w = F_r \cdot R_\delta$$

where R_δ is the static (0 Hz) resistance of a wire having conductive radius r_c equal to the skin depth at the given frequency; $R_\delta = R_w(r_c = \delta)$. Skin depth is a length-constant and is like a spatial time-constant in that it is an exponent in an exponential function of distance. The equivalent area in which the crowded current flows near the wire surface is one *skin depth* δ.

F_r is a mathematically involved function of hyperbolic sines and cosines, but it can be approximated with some simple algebra. What is of interest for this article is that in either the exact or approximate equations, F_r is a function of two parameters: the number of winding layers M, and the *penetration ratio* ξ which for round wire is

$$\xi_r = \frac{r_c}{\delta}$$

Think of ξ_r as the wire size normalized to—or given in units of—the skin depth. With constant frequency, $\delta(f)$ is constant and ξ_r varies with wire size. Then $F_r = F_r(\xi_r, M)$ and is usually plotted with ξ_r on the horizontal axis and M as a parameter. Thus, each of the curves in a family of F_r plots holds M constant for each plot. The result is shown in the graph of Fig. 1 for M = 1 to 8. The lowest (solid) plot is that of F_{rw}, the isolated-wire case.

In Fig. 1, a constant frequency is assumed in these calculations, but since the frequency is not specified in this case, F_r is plotted against units of skin depth rather than wire gauge. If these curves were plotted at a specific frequency versus wire gauge, the curves would look similar to those shown but the wire gauge values would shift left or right depending on the frequency. Some examples are shown in reference 1.

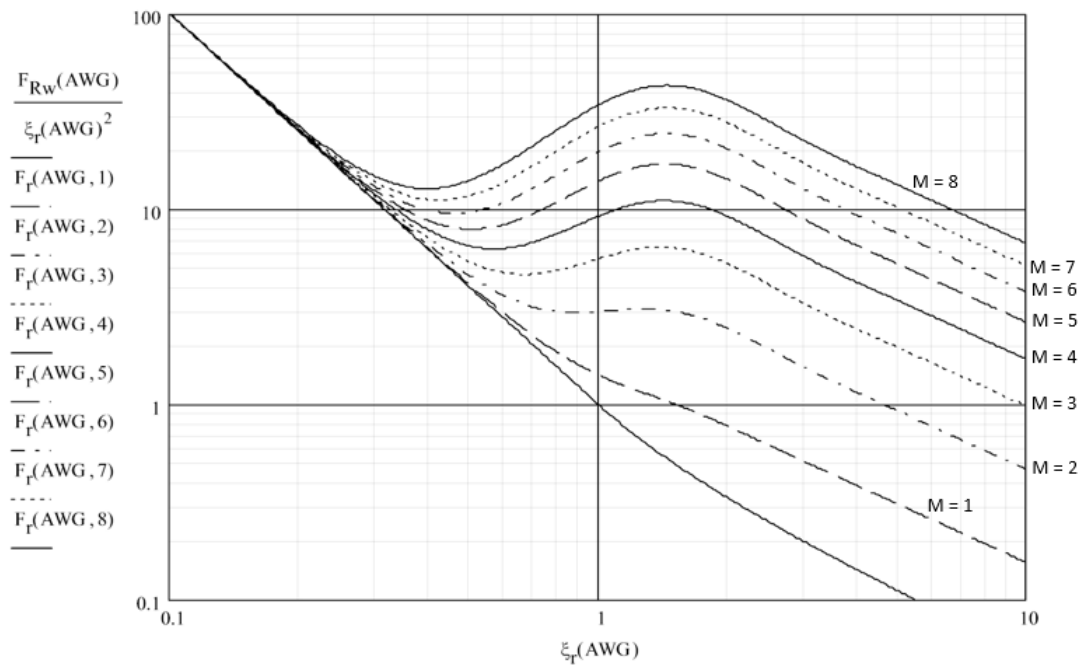

Fig. 1. Log-log graph of constant-frequency resistance ratio F_r, the increase of round-wire resistance from eddy currents at a fixed frequency over resistance at a constant (static) current. F_r is plotted against ξ_r, the round-wire conductive radius in the unit of skin depths at the given frequency. Each different plot has a constant number of winding layers M from 1 to 8. The bottom (solid) plot is that of a single isolated wire.

Eddy-current effects have two contributions which add: the *skin effect*, shown by the lowest isolated-wire plot, and the effect that both adjacent turns within a layer and layers of windings have on each other: the *proximity effect*. This second contributor is like an inter-wire skin effect that causes F_r to rise in the medium-ξ_r region of the plots. The more layers, the higher is the resistance caused by the proximity effect.

The plots do not intersect. However, at very low ξ_r, the plots converge asymptotically to a slope of –2 (on the log-log plot) so that they all have about the same high resistance. These tiny wires, however, do not take up much space in a winding window, and many of them can be paralleled as *strands* of a bundle to increase current capability, or *ampacity*. This is *Litz wire*: bundled strands of smaller wires.

F_r decreases for large-size wire because the equivalent ring of δ thickness at its periphery also expands in cross-sectional area with expanding circumference. However, if too many turns are required (to keep from overheating the core) then the big wire might not fit the core winding window.

Finding Optimal Wire Size With Wire Bundling

A related aspect of winding design is the *geometry* of the winding(s). An insulated wire with an insulated radius of r_{cw} occupies $2 \cdot r_{cw}$ of width along the winding window. For a window of width w_w, then the maximum number of turns per layer is

$$N_l = \frac{w_w}{2 \cdot r_{cw}}$$

For a winding of N_b bundle turns with each turn being a bundle of N_s strands, then the total strands of wire in the window cross-section is $N = N_s \cdot N_b$, and the number of layers for a constant N is

$$M_N = \frac{N}{N_l} = \frac{N \cdot (2 \cdot r_{cw})}{w_w}$$

This can be written by substituting ξ_r of the Dowell graph with one more wire parameter, the *porosity* k_{pw}: the ratio of insulated-wire cross-sectional area A_{cw} to that of the inner conductive area A_c. Then

$$k_{pw} = \frac{A_c}{A_{cw}} = \frac{\pi \cdot r_c^2}{\pi \cdot r_{cw}^2} \Rightarrow r_c = \sqrt{k_{pw}} \cdot r_{cw} \Rightarrow \xi_r = \frac{r_c}{\delta} = \frac{\sqrt{k_{pw}} \cdot r_{cw}}{\delta} \Rightarrow r_{cw} = \frac{\delta}{\sqrt{k_{pw}}} \cdot \xi_r$$

Substituting for r_{cw} in M_N results in a relationship between M_N and ξ_r;

$$M_N = \frac{2 \cdot N \cdot \delta}{\sqrt{k_{pw}} \cdot w_w} \cdot \xi_r$$

The idea emerged that eddy-current plots need not be based only on constant layers M, but that M can be related to the winding geometry by applying to it various constraints. Substituting this geometrically-constrained M into F_r as $F_r(\xi_r, M_N(\xi_r))$, when plotted, appears in Fig. 2 as the blue (dash) line.

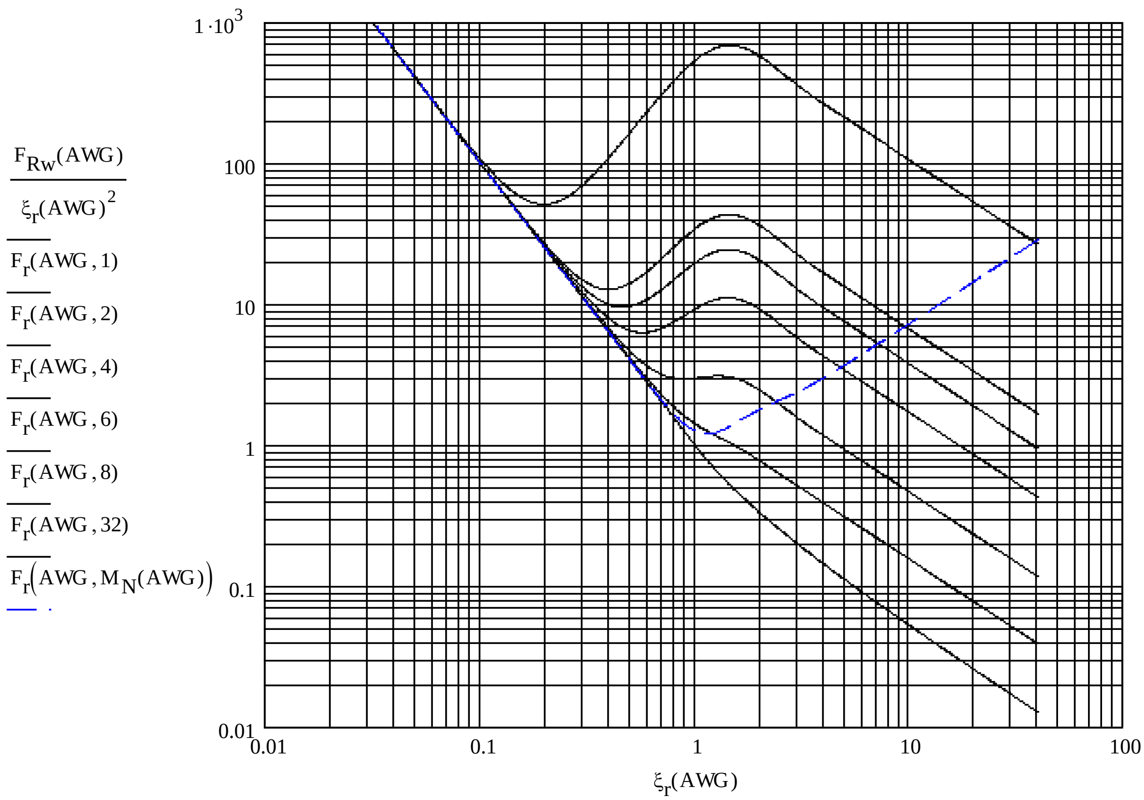

Fig. 2. F_r plots of constant layers M versus wire size in number of skin depths. The blue (dash) line is $F_r(\xi_r, M_N)$, or F_r when M is allowed to vary such that the winding is geometrically constrained to a fixed number of strands per bundle N_s. It has a minimum near $\xi_r = 1$.

To minimize winding loss, the goal is to minimize winding resistance, and this plot clearly reveals a minimum around $\xi_r \approx 1$ for a fixed number of strands per bundle. Knowing the transformer drive frequency f (which for push-pull converters is $f_s/2$), the skin depth can be found from its formula for copper wire;

$$\delta_{\mathrm{Cu}} = \frac{73.5\ \mathrm{mm}}{\sqrt{f\,/\,\mathrm{Hz}}}\ ,\ 80\ {}^{\circ}\mathrm{C},\ \mathrm{Cu}$$

Then the optimal wire size that minimizes F_r is $r_c = \xi_r \cdot \delta_{\mathrm{Cu}}$. Multiple strands of it will usually be needed to achieve the current-carrying capacity (ampacity) required by the circuit design at its maximum winding current.

Finding Optimal Wire Size For A Winding Window Area

A more useful optimization is for a constant winding window. The windings to be fit into the core window are allotted a given area and with some more algebra like that used to derive M_N, the result for a constant area, where k_{ww} is the fraction of window area A_w allotted to the winding ($A_{ww} = k_{ww} \cdot A_w$), then

$$M_A = \frac{2 \cdot k_{ww} \cdot k_p \cdot A_w}{\pi \cdot \sqrt{k_{pw}} \cdot \delta \cdot w_w} \cdot \frac{1}{\xi_r}$$

where k_p is the *packing factor*, the ratio of conductive area in the winding to total area. (k_{pw} is part of k_p; in total, $k_p = k_{pf} \cdot k_{pb} \cdot k_{pw}$ where k_{pf} is the fill factor among the bundles in the winding, k_{pb} is the fill factor of strands within the bundle, and k_{pw} is the porosity caused by wire insulation spacing. Typically, $k_{pf} \approx k_{pb} \approx \pi/4 \approx 0.7845$.) When $F_r(\xi_r, M_A(\xi_r))$ is plotted, the graph of Fig. 3 results.

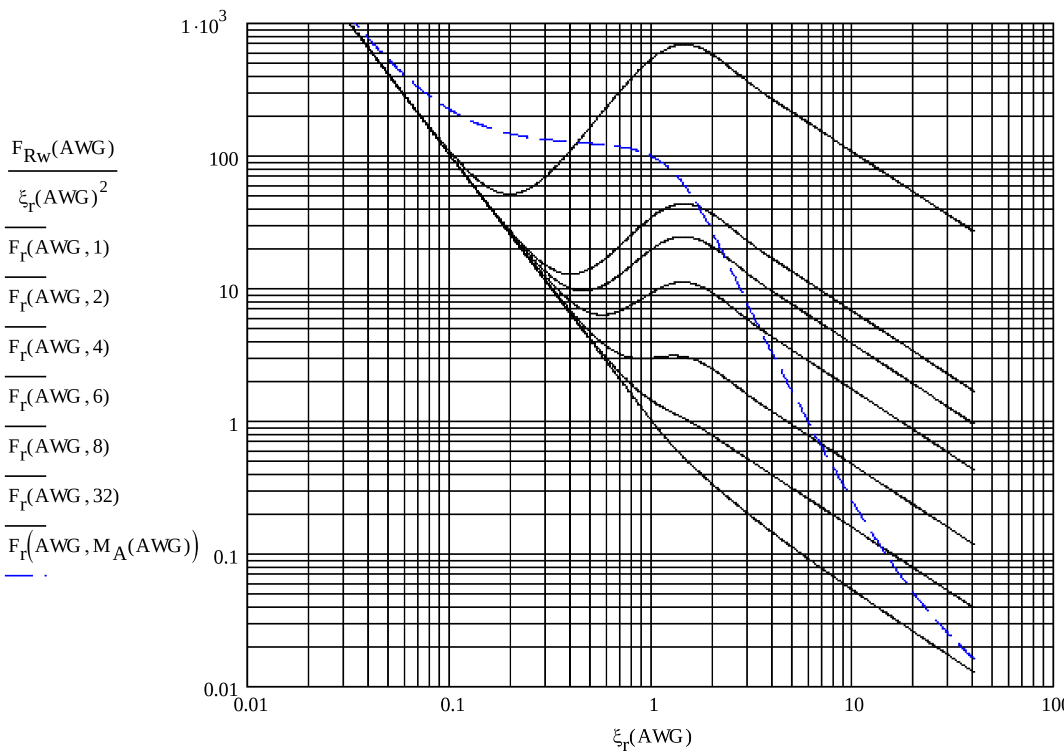

Fig. 3. F_r plots of constant layers M versus wire size in number of skin depths. The blue (dash) line is $F_r(\xi_r, M_A)$, or strand F_r when M is allowed to vary such that the winding is geometrically constrained to a constant winding area A_{ww}. It has no minimum but decreases monotonically with ξ_r. For strands in a bundle, F_r has a different shape.

The blue (dash) plot continues to decrease for increasing wire size (ξ_r), but there is more to the optimization story, to be taken up in the next part of this series. Meanwhile, all of this is developed more comprehensively and in more detail in *Power Magnetics Design Optimization* (*PMDO*).[2] If you want an open-source PDF copy (about 10 Mbytes), contact me and request a PDF copy through the Innovatia website.[3]

As this series continues with two more installments, it will expand on the techniques for using Dowell's equations to determine optimal wire size for a constant number of layers or strands (part 2) and for a constant winding area (part 3.)

References

1. "Eddy-Current Effects In Magnetic Design (Part 5): Winding Design Optimization" by Dennis Feucht, How2Power Today, February 2017.
2. Power Magnetics Design Optimization, Dennis L. Feucht, Innovatia, innovatia.com
3. Contact the author here.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For more on magnetics design, see these How2Power Design Guide search results.