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Non-Inverting Integrators Are Not Really Integrators (Part 1)

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Integrators find use in a huge variety of electronic devices. Some of them use integrators to perform a mathematical operation of integrating analog signals.*

An example of such an integrator application is a Rogowski current sensor. This type of a magnetic sensor utilizes a weak linkage between the ac current carrying bus and a coil placed around this bus that reacts to the magnetic field changes caused by the bus changing current. As the output of the Rogowski coil is voltage that represents the current deviations in the bus, this voltage is proportional to the differentiated current in the bus.

To restore the waveform of the current, an integrator is used following the Rogowski coil. The integrator output voltage is supposed to be a copy of the bus current waveform. Therefore, the integrator should perform its function accurately to restore the shape of the current function.

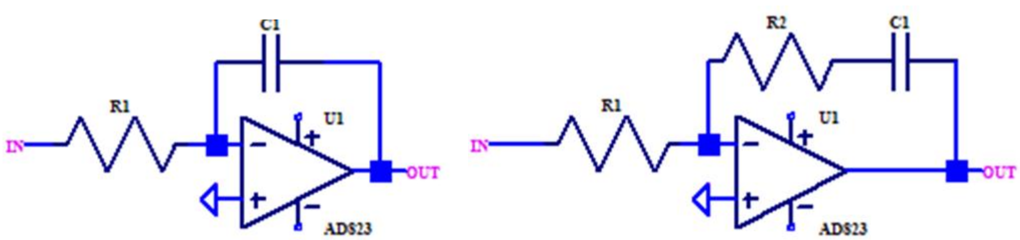
Many authors have presented non-inverting schemes for the integrator. However, these circuits produce erroneous results. Because of the non-inverting input, the transfer function of a non-inverting integrator obtains a zero at the pole frequency, thus destroying the integration function. In this article series, we'll analyze various forms of inverting and non-inverting integrators to confirm this problem and then present examples that illustrate the differences in performance.

Here in part 1, we begin by reviewing and analyzing schematics of integrators with and without phase correction.

**Note: There are also some integrating RC circuits that are used to generate sawtooth waveforms. These are sometimes erroneously referred to as integrators.*

Classical Vs. Phase-Corrected Integrator

Consider the two forms of inverting integrator depicted in Fig. 1.



(a) (b)

Fig. 1. Classical inverting integrator (a) and inverting integrator with phase correction (b).

It is easy to notice that these schematics are dc-unstable since they integrate noise and input bias voltage, which is reflected by the output voltage walkaway. There are two main methods of stabilizing the inverting integrators, and they will be reviewed in later parts of this series.

But ignoring the dc instability issue for now, we see that phase correction is provided by adding resistor R2 in series with C1. This introduces a pole into the transfer function of the integrator thus improving stability of the circuit the integrator is a part of.

To determine the responses of the two inverting integrator circuits, we begin by noting the feedback impedance for each:

$$Z_1(s) = \frac{1}{s \cdot C1} \quad (1a)$$

$$Z_2(s) = R2 + \frac{1}{s \cdot C1} \quad (1b)$$

The integrator transfer functions are therefore:

$$G_1(s) = \frac{\frac{1}{s \cdot C1}}{R1} \quad (2a)$$

$$G_2(s) = \frac{\frac{C1 \cdot R2 \cdot s+1}{C1 \cdot s}}{R1} \quad (2b)$$

Simplifying these expressions we get:

$$G_1(s) = \frac{1}{C1 \cdot R1 \cdot s} \quad (3a)$$

$$G_2(s) = \frac{C1 \cdot R2 \cdot s+1}{C1 \cdot R1 \cdot s} \quad (3b)$$

To obtain the amplitude-frequency characteristic for each integrator we have to get the modulus of the complex frequency response:

$$G_1(j \cdot \omega) = \frac{1}{j \cdot \omega \cdot C1 \cdot R1} \quad (4a)$$

$$G_2(j \cdot \omega) = \frac{C1 \cdot R2 \cdot j \cdot \omega+1}{C1 \cdot R1 \cdot j \cdot \omega} \quad (4b)$$

Then, multiplying both numerator and denominator by the complex conjugate expression, we get rid of the complexity in the denominator:

$$G_1(j \cdot \omega) = \frac{-(j \cdot \omega \cdot C1 \cdot R1)}{(\omega \cdot C1 \cdot R1)^2}$$

$$G_2(j \cdot \omega) = \frac{C1^2 \cdot R1 \cdot R2 \cdot \omega^2 - C1 \cdot R1 \cdot \omega \cdot j}{(C1 \cdot R1 \cdot \omega)^2}$$

Now, define the moduli of both expressions:

$$G_1(\omega) = \frac{\omega \cdot C1 \cdot R1}{(\omega \cdot C1 \cdot R1)^2}$$

$$G_2(\omega) = \frac{\sqrt{(C1^2 \cdot R1 \cdot R2 \cdot \omega^2)^2 + (C1 \cdot R1 \cdot \omega)^2}}{(C1 \cdot R1 \cdot \omega)^2}$$

Simplifying, we obtain:

$$G_1(\omega) = \frac{1}{\omega \cdot C1 \cdot R1} \quad (5a)$$

$$G_2(\omega) = \frac{\sqrt{1+(C1 \cdot R2 \cdot \omega)^2}}{\omega \cdot C1 \cdot R1} \quad (5b)$$

Analyzing expressions (5A) and (5B), we notice that the integration time constants of these expressions are the same—C1R1—and that G2(ω) is not a pure integrator, but rather a low-pass filter. This is the effect of R2.

To illustrate, let's assign some values to the integrator components:

R1 = 100 kΩ

R2 = 0.1 kΩ

C1 = 0.01 μF

Plugging these values into equations for G1(ω) and G2(ω):

$$G_1(\omega) = \frac{1}{\omega \cdot C1 \cdot R1}$$

$$G_2(\omega) = \frac{\sqrt{1+(C1 \cdot R2 \cdot \omega)^2}}{\omega \cdot C1 \cdot R1}$$

yields the plots in Fig 2.

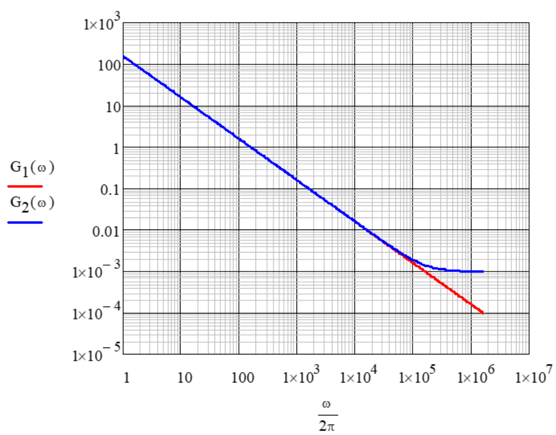


*Fig. 2. We see that in the inverting scheme of the integrator integration begins at 0 Hz, and amplitude-frequency characteristics diverge at the frequency defined by C1*R2, which means we can control content of higher frequencies at the output of the integrator. This is especially useful in Rogowski sensors.*

Now, we have to know the influence of the phase-correcting resistor R2 on the integrating capabilities of the integrator. To do this we can divide one characteristic over another one and check at what frequency the result attains the preset value α. This frequency is defined by the C1*R2 product that allows us to define R2 value at known C1:

Divide (5A) over (5B) and denote the result as α:

$$\frac{\frac{1}{\omega \cdot C1 \cdot R1}}{\frac{\sqrt{1+(C1 \cdot R2 \cdot \omega)^2}}{\omega \cdot C1 \cdot R1}} = \alpha \qquad (6)$$

Simplifying we get:

$$\left(C1^2 \cdot R2^2 \cdot \omega^2 + 1\right)^{\frac{-1}{2}} = \alpha \qquad (7)$$

and solving for ω, we obtain:

$$\begin{bmatrix} \frac{\sqrt{-(\alpha-1)\cdot(\alpha+1)}}{C1 \cdot R2 \cdot \alpha} \\ -\frac{\sqrt{-(\alpha-1)\cdot(\alpha+1)}}{C1 \cdot R2 \cdot \alpha} \end{bmatrix} \qquad (8)$$

The real solution requires:

$$(1-\alpha)\cdot(\alpha+1) > 0 \qquad (9)$$

which yields:

$$-1 < \alpha < 1$$

or rather

$$0 < \alpha < 1 \qquad (10)$$

Assuming

$$\alpha = \frac{\sqrt{2}}{2}$$

which signifies a 3-dB difference, we solve equation (8) for ω:

$$\omega_{01} = \frac{\sqrt{-(\alpha-1)\cdot(\alpha+1)}}{C1 \cdot R2 \cdot \alpha} = 1 \text{ x } 10^6 \frac{1}{s}$$

and

$$\omega_{02} = -\frac{\sqrt{-(\alpha-1)\cdot(\alpha+1)}}{C1 \cdot R2 \cdot \alpha} = -1 \text{ x } 10^6 \frac{1}{s}$$

We can ignore the ω_{02} solution as negative, and therefore if we plug in our example values for C1 and R2, we find

$$f_{01} = \frac{\omega_{01}}{2\pi} = 159.155 \text{ x } 10^3 \frac{1}{s}$$

Therefore, the amplitude-frequency characteristics diverge by 3 dB at 159.2 kHz.

Recalling the expression for ω_{01}

$$\omega_{01} = \frac{\sqrt{-(\alpha-1)\cdot(\alpha+1)}}{C1 \cdot R2 \cdot \alpha}$$

we find R2 for C1 = 1 x 10^{-8} F

$$R2 = \frac{\sqrt{-(\alpha - 1) \cdot (\alpha + 1)}}{C1 \cdot \alpha \cdot \omega_{01}} = 100\ \Omega$$

The phase correction resistor R2 (in further notation R21) depends on coefficient α hyperbolically

$$R21(\alpha 1) = \frac{\sqrt{(1 - \alpha 1) \cdot (1 + \alpha 1)}}{C1 \cdot \alpha 1 \cdot \omega_{01}}$$

as illustrated by the plot in Fig. 3.

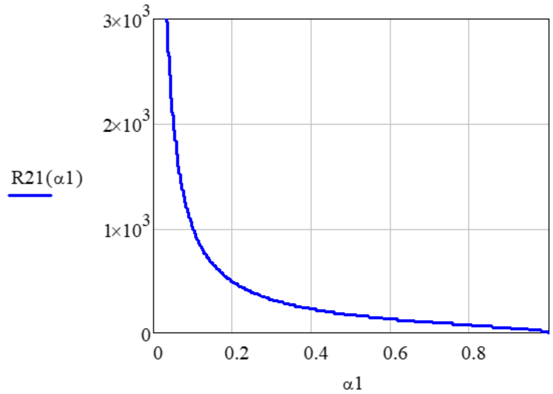


Fig. 3. It is reasonable to select the phase correction resistor value corresponding to α closer to 1 (0.707 or 3 dB is optimal) to keep the integration frequency range wide enough. But too low values of this resistor may not help suppress possible oscillations in the system. There will be further discussion on this topic in future articles.

The Takeaway

As this analysis has demonstrated, the inverting integrator is a real integrator since its amplitude-frequency characteristic begins at zero hertz and extends down to infinity or any value that can be set up by design. The phase correction does not affect this feature. In part 2, we'll analyze the response of a non-inverting integrator.

References

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About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.