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Non-Inverting Integrators Are Not Really Integrators (Part 2)

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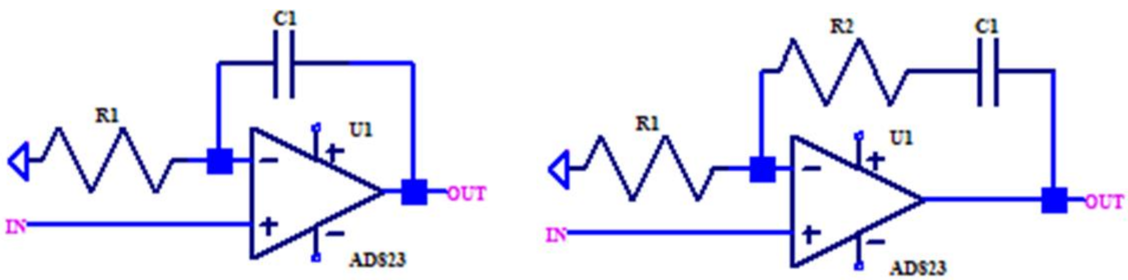
The first part of this series dedicated to different types of integrators described the standard inverting integrator circuit and a variant with phase correction. Through analysis, it was shown that inverting integrators (with or without phase correction) are real integrators because their amplitude-frequency characteristics are down sloping at a constant rate unlimitedly, assuming there are no parasitic parameters.

Now, it is time to review non-inverting integrators, which are widely promoted[1-3] as proposed integrator solutions and draw some conclusions about their integrating abilities.

Non-Inverting Integrators With and Without Phase Correction

Consider the two forms of non-inverting integrator depicted in Fig. 1.



(a) (b)

Fig. 1. A so-called non-inverting integrator (a) and a variation with phase correction (b).

For these circuits, the transfer functions are

$$G_3(s) = 1 + \frac{1}{C1 \cdot R1 \cdot s}$$

Simplifying and factoring yields

$$G_3(s) = \frac{C1 \cdot R1 \cdot s + 1}{C1 \cdot R1 \cdot s} \quad (1a)$$

$$G_4(s) = 1 + \frac{R2 + \frac{1}{s \cdot C1}}{R1}$$

Simplifying and factoring yields

$$G_4(s) = \frac{C1 \cdot R1 \cdot s + C1 \cdot R2 \cdot s + 1}{C1 \cdot R1 \cdot s} \quad (1b)$$

Converting to the frequency domain, these responses become

$$G_3(j \cdot \omega) = \frac{C1 \cdot R1 \cdot j \cdot \omega + 1}{C1 \cdot R1 \cdot j \cdot \omega} \quad (2a)$$

$$G_4(j \cdot \omega) = \frac{[C1 \cdot (R1 + R2)] \cdot j \cdot \omega + 1}{C1 \cdot R1 \cdot j \cdot \omega} \quad (2b)$$

Removing the imaginary expression from the denominators by multiplying them by the complex conjugate we get:

$$G_3(j \cdot \omega) = \frac{(C1 \cdot R1 \cdot j \cdot \omega + 1) \cdot [-(C1 \cdot R1 \cdot j \cdot \omega)]}{(C1 \cdot R1 \cdot \omega)^2} \quad (3a)$$

$$G_4(j \cdot \omega) = \frac{[[C1 \cdot (R1 + R2)] \cdot j \cdot \omega + 1] \cdot [-(C1 \cdot R1 \cdot j \cdot \omega)]}{(C1 \cdot R1 \cdot \omega)^2} \quad (3b)$$

which then become

$$G_3(j \cdot \omega) = \frac{C1^2 \cdot R1^2 \cdot \omega^2 - j \cdot (C1 \cdot R1 \cdot \omega)}{(C1 \cdot R1 \cdot \omega)^2} \quad (4a)$$

$$G_4(j \cdot \omega) = \frac{C1^2 \cdot (R1 + R2) \cdot R1 \cdot \omega^2 - j \cdot C1 \cdot R1 \cdot \omega}{(C1 \cdot R1 \cdot \omega)^2} \quad (4b)$$

The moduli of these expressions are the amplitude-frequency characteristics we seek:

$$G_3(\omega) = \frac{\sqrt{(C1^2 \cdot R1^2 \cdot \omega^2)^2 + (C1 \cdot R1 \cdot \omega)^2}}{(C1 \cdot R1 \cdot \omega)^2}$$

Simplifying and factoring yields

$$G_3(\omega) = \frac{\sqrt{C1^4 \cdot R1^4 \cdot \omega^4 + C1^2 \cdot R1^2 \cdot \omega^2}}{C1^2 \cdot R1^2 \cdot \omega^2} \quad (5a)$$

$$G_4(\omega) = \frac{\sqrt{(C1^2 \cdot R1^2 + 2 \cdot C1^2 \cdot R1R2 + C1^2 \cdot R1^2) \cdot \omega^2 + 1}}{C1 \cdot R1 \cdot \omega} \quad (5b)$$

Simplifying we get the following amplitude-frequency characteristics:

$$G_3(\omega) = \frac{\sqrt{C1^2 \cdot R1^2 \cdot \omega^2 \cdot (1 + C1^2 \cdot R1^2 \cdot \omega^2)}}{C1^2 \cdot R1^2 \cdot \omega^2} = \frac{\sqrt{1 + C1^2 \cdot R1^2 \cdot \omega^2}}{C1 \cdot R1 \cdot \omega}$$

$$G_4(\omega) = \frac{\sqrt{C1^2 \cdot (R1 + R2)^2 \cdot \omega^2 + 1}}{C1 \cdot R1 \cdot \omega}$$

To illustrate the responses of these two circuits, let’s assign component values.

R1 = 100 kΩ

R2 = 0.1 kΩ

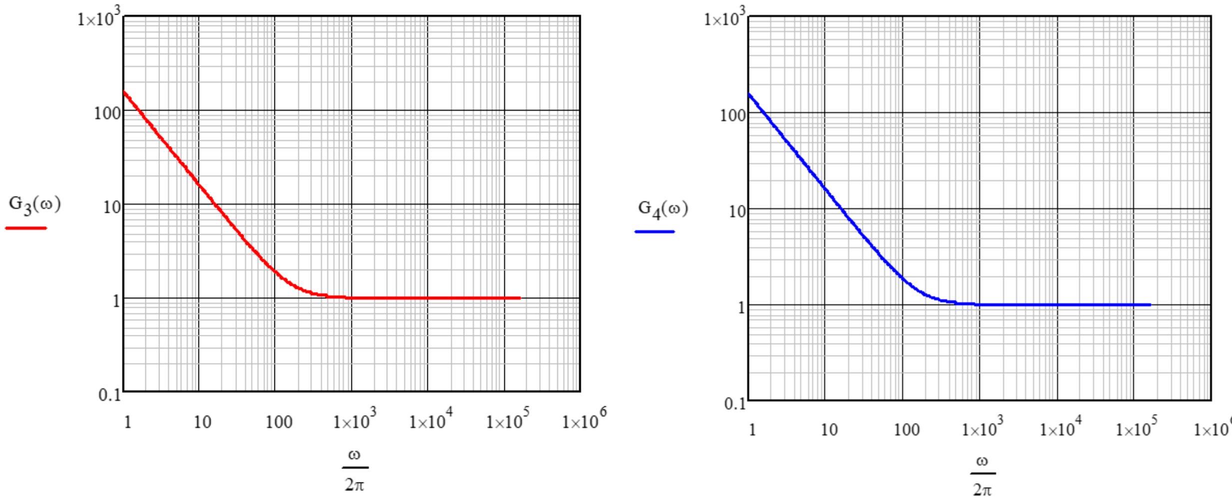
C1 = 0.01 µF

Plugging these values into the expressions for $G_3(\omega)$ and $G_4(\omega)$.

$$G_3(\omega) = \frac{\sqrt{1 + C1^2 \cdot R1^2 \cdot \omega^2}}{C1 \cdot R1 \cdot \omega}$$

$$G_4(\omega) = \frac{\sqrt{C1^2 \cdot (R1 + R2)^2 \cdot \omega^2 + 1}}{C1 \cdot R1 \cdot \omega}$$

produces the responses plotted in Fig. 2.



(a) (b)

Fig. 2. Example plots of responses for the non-inverting integrator (a) and non-inverting integrator with phase correction (b) from Fig. 1.

For reference, recall that the inverting integrators from part 1 had the responses repeated here in Fig. 3.

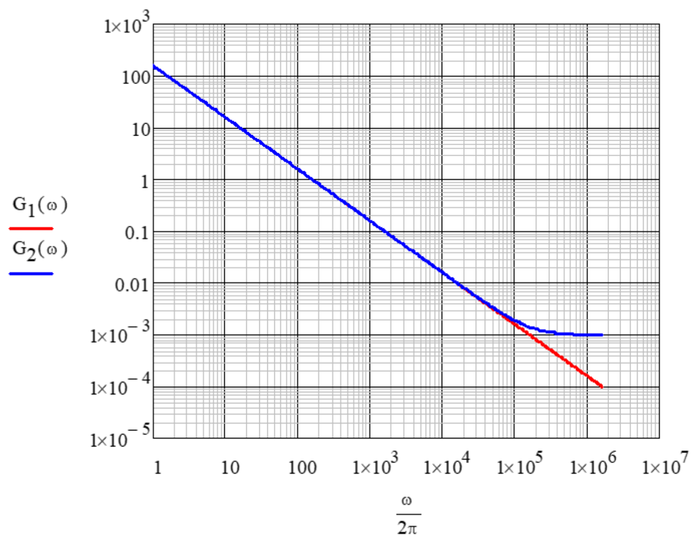


*Fig. 3. With the inverting integrator circuits analyzed previously, integration begins at 0 Hz, and amplitude-frequency characteristics diverge at the frequency defined by C1*R2, which is a point 3 dB below the limit line for the non-inverting circuits.*

The Takeaway

A non-inverting integrator has a very limited area of integration since it has a path of a direct signal pass from the input to the output. Its integration frequency band is narrow since it's limited by the unity gain of the integrator, which stems from the non-inverting connection of the integrator. Therefore, a non-inverting configuration is never the best choice for an integrator.

References

1. "Traditional non-inverting integrator circuits and their corresponding gain-frequency characteristics," ResearchGate website.
2. "Electronic Integrator for Rogowski Coil Sensors," U.S. Patent US 9,588,147 B2.
3. "High Performance Rogowski Current Transducers," Fig. 10, W. F. Ray and C. R. Hewson.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.