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Optimized Magnetics Winding Design (Part 2): Minimized Winding Resistance For Constant Layers Or Strands

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In the previous part of this article,[1] the constant-frequency eddy-current resistance ratio F_r, which is usually plotted with constant M winding layers as a parameter, instead allows M to vary according to various geometric constraints on the winding window such as number of bundle strands or winding cross-sectional area. The equation giving the resistance ratio for windings was constrained by the winding geometry to result in the optimal wire size of the winding of magnetic components.

However, as we discovered in the earlier analysis, making winding area constant did not lead to a clear minimum for F_r and therefore provided no clarity on optimal wire size. In this article, we see how this problem can be overcome by solving for the constant-frequency eddy-current resistance ratio of a bundle (F_r / N_S), rather than that of the individual strands. We then delve further into how this resistance ratio varies for constant number of layers and strands, and in part 3, constant winding area.

Why Analyze By Bundles And Not Strands

Usually the most useful constraint for winding design holds the allotted area of a winding constant. This results in the equation (repeated here from part 1) for number of layers,

$$M_A = \frac{2 \cdot k_p \cdot (k_{ww} \cdot A_w)}{\pi \cdot \sqrt{k_{pw}} \cdot \delta \cdot w_w} \cdot \frac{1}{\xi_r}$$

where w_w = winding (window) width; k_{ww} = fraction of window area allotted to winding; A_w = core window cross-sectional area; k_p = winding packing factor, the fraction of conductor to total area; k_{pw} = wire porosity, the fraction of insulated wire area that is conductor; δ = eddy-current skin depth, and ξ_r is the round-wire conductive radius in number of δ, or $\xi_r = r_c/\delta$. The plot of the constant-frequency resistance ratio from part 1 (Fig. 3) is repeated here as Fig. 1.

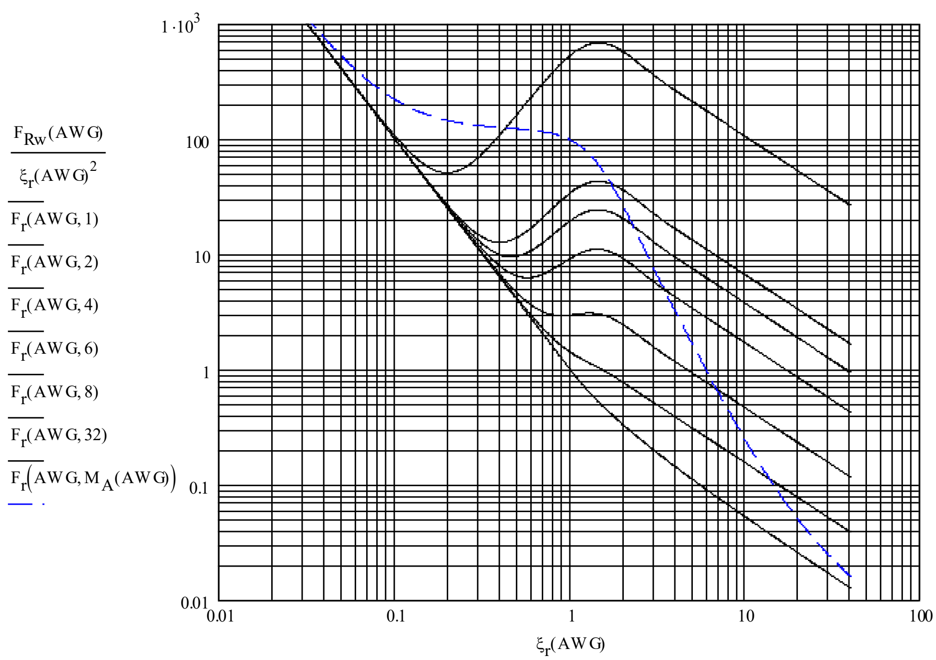

Fig. 1. F_r plots of constant layers, M versus wire size in number of skin depths. The blue (dash) line is $F_r(\xi_r, M_A)$, or strand F_r when M is allowed to vary such that the winding is geometrically constrained to a constant winding area A_{ww}. It has no minimum but decreases monotonically with ξ_r. For strands in a bundle, F_r has a different shape.

In this form, we see no optimal minimum like we did for the constant-strands geometric constraint. However, some further thought about the overall goal—to minimize winding resistance—reveals another insight. From part 1, F_r applies to strands, not the bundle of strands which comprises the winding turns. The goal for design is to minimize winding resistance (referred here to the primary winding, *p*), applied to the bundle as

$$R_{wp} = F_r \cdot \left(\frac{R_\delta}{l_w}\right) \cdot \left(\frac{l_w}{N_s}\right) = \frac{F_r}{N_s} \cdot \left[\left(\frac{R_\delta}{l_w}\right) \cdot l_w\right] = f_r \cdot \left[\left(\frac{R_\delta}{l_w}\right) \cdot l_w\right]$$

where R_δ is the resistance of a wire with $r_c = \delta$, l_w is the winding length, and N_s = number of strands. Now define a somewhat modified F_r, the *strand-normalized* or *bundle* F_r:

$$f_r = \frac{F_r}{N_s}$$

In R_{wp}, (R_δ/l_w) is resistance per length and is fixed and known for a given frequency. The bracketed expression remains nearly constant with r_{cw} or N_s. l_w changes only a few percent with large changes in wire strand size r_{cw}. Consequently, R_w varies mostly with f_r. Then f_r becomes the design parameter to minimize,[2] and $f_r = F_r/N_s$ depends on the eddy-current effects which depend on r_c and M at a fixed frequency.

The plots of $f_{rx} = F_{rx}/N_{sx}$ (where x is N, A, or v) are shown below in Fig. 2. (The plots are based on a Ferroxcube EC35 core with winding parameters: copper, N_s = 5, N_b = 6, k_{pw} = 0.86, and a frequency of 500 kHz.)

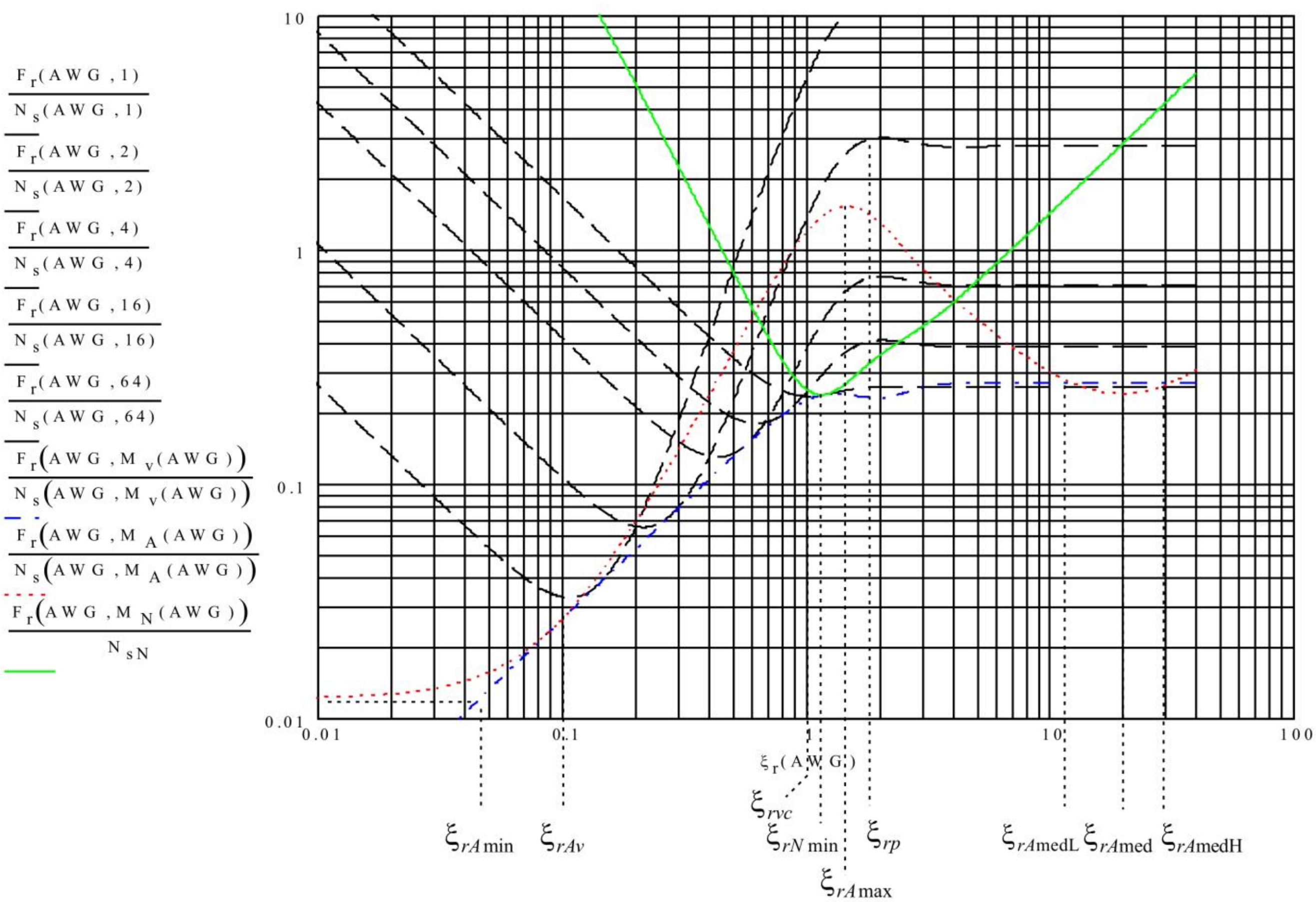

Fig. 2. Bundle F_r, or $f_r = F_r/N_s$ for finding winding-resistance minimums. The black (dash) plots are f_r for constant M. When geometric constraints are applied to the F_r (ξ_r, M) function by allowing M to vary, other plots result: blue (dash-dot) is minimum f_r with variable M; green (solid) is for constant N_s; and red (dot) is for constant winding area.

Plots of the *bundle*-$F_r = f_r = F_r / N_s$ are for F_r minima f_{rv} over the range of M (dash-dot); constant winding area, A_{ww} (dot); and constant strands, N_s (solid), along with the background constant-M plots (dash). Design operating-points of interest are designated by their ξ_r values.

The f_r plot slopes are reduced by one from F_r slopes because N_s causes f_r to vary inversely with ξ_r. The constrained f_{rx} are also affected. The dash plot of f_{rN}, like the constant-strands F_{rN} plots, still decreases to a sharp minimum, then increases. The constant winding-area f_{rA} (red dot plot) at very small strand size (low ξ_r) is asymptotic to a low constant value of $f_{rA\mathrm{min}}$, then breaks around $\xi_{rA\mathrm{min}}$, increases somewhat linearly to a peak somewhat above $\xi_r = 1$ at ξ_{rp}, then decreases to a high-ξ_r minimum, $\xi_{rA\mathrm{med}}$ after which it again increases.

Optimizing For Constant Number Of Layers

On the constant-M F_r plots found in most magnetics textbooks, F_r reaches a minimum before rising from the proximity effect. The minimum or valley values are F_{rv}. Similarly, the minimum-F_r f_{rv} (blue dash-dot plot) for decreasing wire size decreases linearly without bound below a breakpoint at $\xi_r = 1$. Above it, f_{rv} remains close to a constant value designated as

$$f_{rvc} = f_{rv}(\xi_{rvc}) \, , \xi_{rvc} = 1$$

Although f_{rv} does not have any minima above ξ_{rvc}, it serves as a useful reference value to compare to f_{rN} and f_{rA} for magnetic operating-point optimality. At the op-points, the optimal wire radius is calculated from the op-point ξ_r value.

The optimal ξ_r values are found either by plotting them for given magnetic-component parameters or else deriving them analytically as design equations. Analytic formulas are preferred because the Dowell-equation approximations have already been derived analytically.[3] Derivations are based on F_r approximation of the Dowell equation for low ξ_r, of $\xi_r < 1$:

$$F_r(\xi_r, M) \approx \frac{1}{\xi_r^2} + \left(\frac{5 \cdot M^2 - 1}{45}\right) \cdot g_r^4 \cdot \xi_r^2 \;,\; \xi_r < 1 \,,\; g_r \approx 1.547 \text{ for round wire}$$

where g_r is a geometric constant related to porosity and is about 1.55 for round wire. Design formulas for optimal operating-points are found by setting derivatives to zero or equating plot intersections and solving for ξ_r. f_{rv} for a constant M is the simplest. f_{rv} breaks at ξ_{rvc}. For $\xi_r > \xi_{rvc}$, f_{rv} is nearly constant. The f_{rv} curve imposes no lower limit on R_w with decreasing wire size. The low-ξ_r approximation of F_{rv} for $M \geq 2$ is found from the derivative of the low-ξ_r F_r approximation, where

$$\xi_{rv} \approx \frac{1}{g_r} \cdot \left(\frac{45}{5 \cdot M^2 - 1}\right)^{1/4} \approx \frac{1.674}{\sqrt[4]{5 \cdot M^2 - 1}} \approx \frac{1.120}{\sqrt{M}} \;,\; M \geq 2$$

Substituting ξ_{rv} into F_r,

$$F_{rv}(M_v) \approx 2 \cdot g_r^2 \cdot \sqrt{\frac{5 \cdot M_v^2 - 1}{45}} \approx 2 \cdot g_r^2 \cdot \left(\frac{M_v}{3}\right) \;,\; \xi_r < 1, M_v \geq 2$$

where M_v is M at the minimum of $F_r = F_{rv}$. As M increases, so does the accuracy of the equation. At $M = 2$, accuracy is acceptable for most magnetics designs.

To find f_{rv}, substitute ξ_{rv} and N_s (for constant turns per layer, not toroids), and F_{rv} into f_{rv};

$$f_{rv} = \frac{F_{rv}}{N_s} \approx \frac{2 \cdot g_r^2 \cdot \left(\frac{M_v}{3}\right)}{\frac{M_v \cdot w_w \cdot \sqrt{k_{pw}}}{N_b \cdot 2 \cdot \delta} \cdot \frac{1}{\xi_r}} \quad , \xi_r < 1, M \geq 2$$

This reduces to

$$f_{rv} = \frac{4 \cdot g_r^2 \cdot N_b \cdot \delta}{3 \cdot w_w \cdot \sqrt{k_{pw}}} \cdot \xi_r = f_{rvc} \cdot \xi_r \, , \xi_r < 1, M \geq 2,$$

$$f_{rvc} = \frac{F_{rvc}}{N_s} \approx \frac{F_{rv}}{N_s}(\xi_r = 1) = \frac{4 \cdot g_r^2 \cdot N_b \cdot \delta}{3 \cdot w_w \cdot \sqrt{k_{pw}}}$$

At the upper limit of the low-ξ_r F_r approximation, where $\xi_r = 1$, the value (using the EC35 parameters) of $f_{rv}(\xi_r = 1) \approx 0.229$. The f_{rv} plot above, based on Dowell's equation, shows this value to be in agreement for the flat part of the plot. At $\xi_r = \xi_{rvc}$ the plot breaks and f_{rv} is then approximately flat above ξ_{rvc}.

Optimizing For Constant Number Of Strands

The optimization for constant strands seeks $F_{rN\min}(\xi_{rN\min})$ where F_{rN} is minimum. From the above graph, R_w for constant $N_s = N_{sN}$ is minimum at $\xi_r = \xi_{rN\min}$. f_{rN} decreases for increasing $\xi_r < \xi_{rN\min}$. Above $\xi_{rN\min}$ and below ξ_{rp}, F_{rN} rises by r_c^2, and f_{rN} and k_{ww} increase for low-ξ_r. In the high-ξ_r region (above ξ_{rp}), f_{rN} rises with a log-log slope of +1.

To derive $F_{rN\min}(\xi_{rN\min})$, first find F_{rN} by substituting the linear-layered M_N into the F_r approximation. It reduces to

$$F_{rN}(\xi_r) \approx \frac{1}{\xi_r^2} + \left(\frac{g_r^2}{3} \cdot \frac{N \cdot (2 \cdot \delta)}{w_w \cdot \sqrt{k_{pw}}}\right)^2 \cdot \xi_r^4 - \frac{g_r^4}{45} \cdot \xi_r^2 \, , \quad \xi_r < 1$$

The minimum is found by differentiating and solving. For ξ_r not much less than 1,

$$\xi_{rN\min} \approx \sqrt[3]{\frac{\sqrt{2}}{N_s \cdot f_{rvc}}} \approx \frac{1.1225}{\sqrt[3]{N_s \cdot f_{rvc}}}$$

Finally,

$$f_{rN} = \frac{F_{rN}}{N_s} = \frac{1}{N_{sN} \cdot \xi_r^2} + \frac{N_{sN} \cdot f_{rvc}^2}{4} \cdot \xi_r^4 - \frac{g_r^4}{45 \cdot N_{sN}} \cdot \xi_r^2$$

The first term dominates and thus approximates f_{rN} for $\xi_r < \xi_{rN\min}$. The last two terms dominate for $\xi_r > \xi_{rN\min}$. Substituting the EC35 graph parameters, $r_c = \xi_{rN\min} \cdot \delta = (1.031) \cdot (0.1039 \text{ mm}) = 0.1071$ mm, where

$$\delta_{\text{Cu}} = \frac{73.5 \text{ mm}}{\sqrt{f / \text{Hz}}} = \frac{73.5 \text{ mm}}{\sqrt{500 \text{ kHz} / \text{Hz}}} = 0.1039 \text{ mm}$$

From an AWG wire table, the closest size is #31 AWG. Checking,

$$f_{rN} = \frac{1}{5\cdot(1.031)^2} + \left(\frac{2\cdot(1.547)^2\cdot 16\cdot\sqrt{5}\cdot(0.1039\text{ mm})}{3\cdot(21.5\text{ mm})\cdot\sqrt{0.86}}\right)^2\cdot(1.031)^4 - \frac{(1.547)^4}{5\cdot 45}\cdot(1.031)^2 = 0.261$$

This value agrees with $f_{rN\min}$ as read from the f_r graph of Fig. 2, based on the exact Dowell equation. $f_{rN\min}$ is typically around $\xi_r = 1$. $f_{rN\min} \approx f_{rv}\,(\xi_{rN\min})$ and is an optimal operating-point given N_s, subject to the winding fitting within its allotted area. Substituting $\xi_{rN\min}$ into f_{rN} and reducing,

$$f_{rN\min} = \frac{3}{2}\cdot\left(\frac{f_{rvc}^2}{2\cdot N_{sN}}\right)^{1/3} - \frac{g_r^4}{45}\cdot\left(\frac{2}{N_{sN}^5\cdot f_{rvc}^2}\right)^{1/3} \approx \frac{3}{2^{4/3}}\cdot\left(\frac{f_{rvc}^2}{N_{sN}}\right)^{1/3} \approx 1.191\cdot\left(\frac{f_{rvc}^2}{N_{sN}}\right)^{1/3}$$

In comparison,

$$f_{rv}(\xi_{rN\min}) \approx \sqrt[6]{2}\cdot\left(\frac{f_{rvc}^2}{N_{sN}}\right)^{1/3} \approx 1.122\cdot\left(\frac{f_{rvc}^2}{N_{sN}}\right)^{1/3}$$

For the EC35 parameters of the graphs, $f_{rvc} \approx 0.229$, $f_{rN\min} \approx 0.231$, and $f_{rv}(\xi_{rN\min}) \approx 0.246$ and is about 6.3% higher than $f_{rN\min}$. $f_{rN\min}$ and f_{rv} at $\xi_{rN\min}$ are not equal but are close. Thus, $f_{rN\min}$ is close to the f_{rv} optimum.

In the third part of this series, the design formulas for the most useful case of constant winding area are presented. This series is derived from the book *Power Magnetics Design Optimization* (*PMDO*) for which free open-source PDF copies are available by request.

References

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2. "A New Method Of Winding Design Optimization (Part 2): Optimal Window Magnetic Operating Points" by Dennis Feucht, How2Power Today, Oct 2017,
3. Derivations are in *Power Magnetics Design Optimization* (*PMDO*) at www.innovatia.com. PDF copies of *PMDO* are open-source and available by request from the email address at the website.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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