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Motor Control For Designers (Part 1): Basic Principles Of Motor Theory

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With the rising popularity of electronically commutated motors such as brushess dc (BLDC) motors, permanent magnet synchronous motors (PMSM) and variable reluctance (VR) motors across a range of applications, requirements for motor control and motor drive design are growing. For electronics engineers seeking proficiency in these topics, it's necessary to learn not only how the controllers and inverters operate, but also how the motors themselves work. Likewise, motor designers may benefit from an understanding of how the circuits driving the motors function.

This article kicks off a mini-course on motors and circuits that aims to inform both the circuit and the motor designers by covering the following topics: 1) motor theory, 2) motor design, 3) motor-drive design, and 4) motion control. As a single article would be insufficient to adequately introduce any of these topics, each will be addressed in an article series containing about 5 to 8 articles. The goal in publishing these four series—which will be presented in the stated order with all parts numbered sequentially according to their appearance in the overall series or mini-course—is to convey a working design knowledge of electric machines and the electronics and software to control them.

With regard to the first subject, various textbooks on motors have existed for decades and I will share my recommendations on those I consider most useful for learning motor theory. However, while those texts are mainly oriented to analysis, this mini-course will be *design-oriented*.

In this part 1 article, I'll be introducing the principles of motor theory, identifying the key terms, concepts and relationships in electricity and magnetism that are needed to explain how motors work. I'll set the table for this discussion by briefly reviewing the development of motor theory. But before delving further into this discussion of motor principles, I'll say more on the scope of the first two series within this mini-course.

What Will Be Covered

In presenting the four series on motor theory, motor design, motor-drive design, and motion control, the emphasis in this mini-course will be on permanent-magnet synchronous (PMS) motors with peripheral attention given to variable-reluctance (VR) motors and none to induction motors. Development is for rotational and not translational motors, although it is shown how conversion of the equations is rather simple.

The series on motor theory will cover the fundaments of motors, starting with what they are mechanically, and then present derivations of how motors with teeth and windows in their stators develop torque. This series will address various basic aspects of motors: basic E & M (part 1), the torque-current relationship from the Lorentz force equation, then from field energy; inner and outer rotors, pole-pairs, phase-windings and their configurations, their induced-voltage waveforms, and how to sequence drive to them, and then the important electrical-mechanical-analogy model of a motor. Then follows motor performance shown by the (quasistatic) torque-speed graph and the properties of $T(\omega_{me})$.

The second series, on motor design, will be approached from a motor-drive designer viewpoint, showing how the torque-speed curve is modified by mechanical geometric (radius, length, magnet thickness) and electrical (turns and wire size) changes in the motor. The series will go a little deeper than this, deriving the optimal width of the air gap between rotor and stator and how the air gap affects motor behavior. This knowledge will give power-circuits engineers deeper insight into the load of their motor-drive circuitry. The drive will usually have feedback and the motor is within the feedback loop. Therefore the motor properties must be known and optimized to do control design.

The Origins Of Motor Theory

Various textbooks on motors have existed for decades. Electric machines courses are typically found in the electrical engineering curricula of engineering schools. Historically, motor theory was worked out by Karl August

Rudolph Steinmetz (Charles Proteus Steinmetz), a poor, hunchbacked German immigrant passing through Ellis Island in the early 20th century.

His theoretical acumen became known to people at General Electric (GE) while he was employed by a small electric company whose owner took him in at the time of his immigration. Steinmetz's loyalty to the owner for helping him at such a critical time required that GE buy the entire company to obtain Steinmetz. It was Steinmetz who worked out the electrical theory, using steady-state phasor theory, to explain Nikola Tesla's induction motor invention.

During the first half the of the twentieth century, big advances in the understanding of motors occurred about once a decade, originating from GE motor research in Schenectady, NY, until the final refinements were made by Purdue professor Paul Krause, also from GE, in the 1960s. Another GE master of motors, Tom Lipo, became a professor at the U. of WI at Madison, another major motor school. (My early mentor, Allan Plunkett, also came from GE.) It can truly be said that knowledge of motors in the world today emanated out of Steinmetz's work at GE.

Early motors were applied as mechanical power sources and run at a constant speed. This steady-state operation can be understood with fixed-frequency phasor theory found in passive-circuits books, though a more complete theory involving general motion uses vectors. Even today, many of the motor books—maybe the one you used in school—are still based on steady-state phasors.

The book I recommend is that of Krause and Wasynczuk, *Electromechanical Motion Devices* (McGraw-Hill) because it develops the more general vector theory. (The Purdue U. armory might still have some copies of the most rigorous "gold standard" book, *Analysis of Electric Machinery* by Krause, in paperback form.)

What is presented here is a somewhat simplified treatment about *how* to design. Electric machine theory is considered a difficult topic, but that might be largely a result of a lack of clear and simple understanding of it among its practitioners. The treatment developed here keeps it simple; algebra and some basic calculus are all that are needed, plus some understanding of basic mechanics. *Electric machines* are electrical ↔ mechanical energy conversion devices by definition. All of them convert in both directions; all motors are generators.

There are three basic ways of producing torque in (rotational) motors and this series will involve two of them—synchronous and variable-reluctance (VR) machines—and will omit induction machines. Electric machines can mechanically operate in translation (as do levitated trains) or rotation. Rotation is more common and is chosen for these series. Conversion between them is fairly simple, that of moving radius or distance r around in equations. To illustrate, for right angles, torque $T = r \cdot F$ and power $P = T \cdot \omega = (r \cdot F) \cdot (u/r) = F \cdot u$ where ω is rotational speed and u is $r \cdot \omega$ and is translational speed. T and ω are rotational variables and F and u are translational or linear.

Permanent-magnet synchronous (PMS) motors have risen to a place of prominence, found in consumer items, hard disk-drives, scattered about within cars and trucks, and are prominent in electric vehicles. VR machines are also on the rise.

Both PMS and VR machines have a major advantage (as do induction motors) over "dc brush" motors and wound-rotor synchronous motors; they do not have moving electrical contacts in the form of slip rings or brushes—components that are inherently unreliable and subject to wear. With permanent magnets (PMs), there is no need to deliver current to the rotor, nor do VR machines have rotor currents.

Electric machines couple their electrical and mechanical sides through either a magnetic or electric field. For the ranges of voltage and current typical in electronics today, magnetic materials such as electrical steel or ferrites are capable of a higher energy density than dielectric materials as found in capacitors. Consequently, the focus in this series is on electromagnetic energy conversion.

A capable power-electronics engineer can optimize the design of both power circuits and their magnetic components to achieve an overall optimal design. Likewise, good motor-drive designers have an understanding of the "magnetic component"—the motor itself—in the design of a motor-drive.

That is why the second series of this mini-course covers some basic aspects of motor design. How do motor characteristics change when windings have more turns of smaller wire, or the geometry is changed? And how does that optimally couple to both motor-drive electronics and the mechanical load? These questions require design knowledge of both power electronics and motors.

Quantities Of Electricity And Magnetism

This series starts with the most basic quantities and their units, in quick review, given in Table 1.

Table 1. Quantities of electricity and magnetism.

Electricity	**Field-Circuit Relationships**	**Magnetism**
Charge, q in coulombs, C		Field flux, ϕ in V·s
Electric field intensity (strength), E in V/m		Magnetic field intensity (strength), H in A/m
Electric field density, D in C/m^2		Magnetic field density, B in V·s/m^2
Permittivity, ε in F/m		Permeability, μ in H/m
Capacitance, C in farads, F		Inductance, L in henrys, H
	(Circuit-referred) current, i in amperes, A = C/s	λ = circuit-referred flux = $N \cdot \phi$ in V·s
	Current density, J in A/m^2	
	Conductivity, σ in $(\Omega \cdot \text{m})^{-1}$	
$D = \varepsilon \cdot E$	$J = \sigma \cdot E$	$B = \mu \cdot H$
Electric potential, voltage, V in volts, V	Resistivity, $\rho = 1/\sigma$ in $\Omega \cdot$m Area, A in m^2	Magnetic scalar potential (field-referred current), $N \cdot i$ in amperes, A
$v = E \cdot l$	l = length in meters, m	$N \cdot i = H \cdot l$
$C = q/v$	Conductance, $G = \sigma \cdot A/l$	$L = \lambda/i$
$C = \varepsilon \cdot A/l$	Circuit-referred $L = N^2 \cdot$L	Field inductance L = single-turn $L = \mu \cdot A/l$
$i = C \cdot dv/dt$	$i = G \cdot v$ $v = i \cdot R$	$v = L \cdot di/dt$
$q = D \cdot A$		$\phi = B \cdot A$
Electric energy, $W = C \cdot v^2/2 = q \cdot v/2$		Magnetic energy, $W = L \cdot i^2/2 = \lambda \cdot i/2$

Electric and magnetic quantities appear in equations and when voltage and current are exchanged in them, the resulting equations apply to the other, and the quantities are *duals*. For instance, capacitance C and inductance

L are duals because exchange of *v* and *i* in their *v*-*i* equations results in an equation for the other. The middle column of Table 1 relates electric and magnetic field quantities to circuit quantities at the circuit terminals.

Magnetic Reference Frames

Because of winding turns *N*, the effect is to "amplify" circuit current as it appears in the magnetic-field path and induced voltage as it appears across the winding terminals in the circuit. Circuit quantities relate to winding terminal voltages and currents through the referral factor *N* and field quantities corresponding to them relate to the field. Consequently, circuit current *i* appears to the field as *field current* $Ni = N \cdot i$. Winding terminal voltage *v* is referred to the field as induced per-turn voltage v/N or *field voltage*. The field resistance refers to the circuit resistance as field voltage v/N over field current $N \cdot i$ as

$$R_{fld} = \frac{v/N}{N \cdot i} = \frac{v/i}{N^2} = \frac{R_{ckt}}{N^2}$$

The corresponding field resistance is lower than the circuit resistance by N^2. Table 2 summarizes referrals.

Table 2. Circuit and field referrals of corresponding quantities.

Reference-Frame	Current	Inductive quantity	Field quantity	Voltage
Electrical circuit (terminal quantities)	i	Inductance, L	Flux $\lambda = N \cdot \phi$	v
Magnetic field	Field current $Ni = N \cdot i$	Field inductance (permeance) $\mathcal{L}$	Field flux ϕ	Field voltage v/N

Maxwell's Equations

Three of the four basic equations of Maxwell find use in motor theory in a simplified form with right-angle geometries in motors.

Faraday's Law

To begin, let's consider an expression of Faraday's Law:

$$\oint_C \boldsymbol{E} \bullet d\boldsymbol{l} = -\frac{\partial}{\partial t} \int_S \boldsymbol{B} \bullet \hat{\boldsymbol{n}} \cdot da$$

This expression means that a changing magnetic flux $B \cdot A$ through an open surface *S* of cross-sectional area *A* of a magnetic path induces voltage $E \cdot l$ into an electric conductor bounding that area (a winding), with closed (but not shorted) wire path of curve *C*. The component of *field flux* normal to the open surface is $\phi = B \cdot A$.

Considering only this right-angle field flux, Faraday's Law can be reduced to

$$\text{Simplified form: } v = -\frac{d\lambda}{dt} \text{ where } \lambda = N \cdot \phi = N \cdot B \cdot A \text{ and } v = E \cdot l$$

where v = induced voltage, λ = circuit-referred flux, ϕ = field-referred flux, *N* = turns, *B* = magnetic field density (amplitude), *A* is the area through which ϕ is bounded and *l* is the length of the winding path bounding *A*; *E* is the electric-field intensity, inducing a voltage into the winding surrounding the magnetic path of *B*.

Ampere's Law

Next, let's consider a form of Ampere's Law:

$$\oint_C \boldsymbol{H} \bullet d\boldsymbol{l} = \int_S \boldsymbol{J} \bullet \hat{\boldsymbol{n}} \cdot da + \frac{\partial}{\partial t} \int_S \boldsymbol{D} \bullet \hat{\boldsymbol{n}} \cdot da$$

This expression can be interpreted as follows: A field-referred current $J \cdot A$ plus a changing electric-field flux $D \cdot A$ though an open surface S of cross-sectional area A gives rise to a field-referred current $H \cdot l$ in a closed path, curve C, bounded by A having boundary length l. The component of $Ni = J \cdot A$ and $q = D \cdot A$ through (normal to) the open surface S is $H \cdot l$. Right-angle geometry reduces Ampere's Law to

Simplified form: $H \cdot l = N \cdot i + \frac{\partial q}{\partial t}$ where $q = D \cdot A$ and $N \cdot i = J \cdot A$

The second term, $\partial q/\partial t$, is the *displacement current* which is insignificant in high-resistance magnetic materials. Hence, the magnetic field intensity H around the closed magnetic path of length l is produced by a field current $Ni = N \cdot i$. The circuit current i at the terminals of a winding with N turns appears in the magnetic path as *field current*.

Gauss's Electric Law

The electric flux from charge out of closed surface S equals the charge in the volume S encloses;

$$\oint_S \boldsymbol{D} \bullet \hat{\boldsymbol{n}} \cdot da = \int_V \rho \cdot dv$$

This equation would apply to electrostatic motors (as in the ophthalmic probes of phacomachines of eye cataract surgery) but we will not need it for electromagnetic motors. Nevertheless, considering only the flux normal to the surface, Gauss's Electric Law reduces to

Simplified form: $D \cdot A = q$

Gauss's Magnetic Law

Finally, we come to an expression of Gauss's Magnetic Law:

$$\oint_S \boldsymbol{B} \bullet \hat{\boldsymbol{n}} \cdot da = 0$$

The net magnetic flux out of a closed surface is zero. Flux exiting a closed surface must equal the flux entering it, resulting in closed magnetic paths. This applies as much to magnetic fields in motors as in transformers. Again, considering only the flux normal to the surface, leads to

Simplified form: $\phi = B \cdot A = 0$, closed surface (closed path)

Magnetic-Electric Analogs

Current in circuits travels in closed paths as do magnetic fields in magnetic circuits. Because of electric and magnetic circuit *duality* there are corresponding magnetic laws to Kirchhoff's current and voltage laws. Table 3 summarizes the duality.

Table 3. Electric and magnetic circuit analogs.

Electric circuit quantity	Magnetic circuit quantity
Current, i	Magnetic field flux, ϕ
Voltage, v	Field current, $Ni = N \cdot i$
Conductance, G	Field inductance, $\mathcal{L}$

where $\mathcal{L}$ is *field inductance*, the per-turn-square inductance of a single turn. (In magnetics catalogs, $\mathcal{L} \equiv A_L$.) There is no capacitance analog for this aspect of inductors, and the full equation relating circuit and field inductance is

$$L = N^2 \cdot \mathcal{L}$$

Magnetic Ohm's Law

The "Ohm's Law (ΩL)" of magnetic closed-loop circuits is

$$\phi = \mathcal{L} \cdot (N \cdot i) \quad \text{M}\Omega\text{L}$$

MΩL is analogous to Ohm's Law when expressed using conductance:

$$i = G \cdot v \qquad \Omega\text{L}$$

MΩL refers to the electric circuit as circuit quantities, where magnetic field flux ϕ is referred to the electric circuit by *N* as *circuit flux*

$$\lambda = N \cdot \phi = (N^2 \cdot \mathcal{L}) \cdot i = L \cdot i$$

Magnetic Kirchhoff's Laws

The electric-circuit KCL is the familiar

$$\sum_{node} i = 0 \qquad \text{KCL}$$

From Gauss's Magnetic Law,

$$\sum_{node} B \cdot A = \sum \phi = \sum \mathcal{L} \cdot (Ni) = 0 \qquad \text{MKCL}$$

And the circuit KVL is

$$\sum_{loop} v = 0 \qquad \text{KVL}$$

From Ampere's Law,

$$\sum_{loop} H \cdot l = N \cdot i$$

This can be expressed in ϕ and $\mathcal{L}$ as

$$\sum_{loop} \left(\frac{B}{\mu}\right) \cdot l = \sum \frac{B \cdot A}{\left(\frac{\mu \cdot A}{l}\right)} = \sum \frac{\phi}{\mathcal{L}} = N \cdot i$$

The relationship between *B* and *H* is $B = \mu \cdot H$. Applying MΩL, the last equation is

$$\frac{\phi}{\mathcal{L}} = N \cdot i$$

The sum of the magnetic $\phi/\mathcal{L}$ drops around the magnetic loop must equal the $N{\cdot}i$ source on the right side of the equation. Therefore

$$\sum_{loop} N \cdot i = 0 \qquad \text{MKVL}$$

In a motor with a magnetic-steel stator core in series with an air gap,

$$N \cdot i = \left(\frac{\phi}{\mathcal{L}_{\text{core}}} + \frac{\phi}{\mathcal{L}_{\text{gap}}} \right)$$

With magnetic analogies of ΩL, KCL, and KVL, and with the three basic equations of Maxwell, we are ready to proceed with motor theory.

Inductance From Geometry

To prepare for motor magnetics, we work through an example problem in finding inductance. See Fig. 1. Assume the magnetic field **B** is parallel to the length axis *l* within the solenoid.

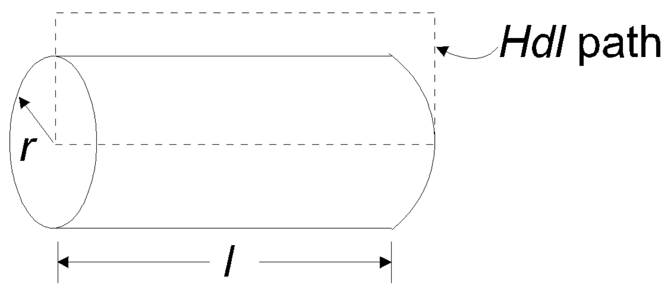

Fig. 1. Long solenoid with r << l, to approximate the magnetic path as shown.

Furthermore, assume $r << l$. Then for N turns around the solenoid of radius r,

$$L = \frac{d(N \cdot \phi)}{di} = N \cdot \frac{d\phi}{di}$$

$$\phi = B \cdot A$$

$$H{\cdot}l = N{\cdot}i$$

Substitute for H in the expression for B:

$$B = \mu \cdot H = \frac{\mu \cdot (N \cdot i)}{l}$$

And substitute B into the flux equation,

$$\phi = \left(\mu \cdot \frac{N \cdot i}{l} \right) \cdot A$$

Then

$$L = N \cdot \frac{d\phi}{di} = N^2 \cdot \left(\frac{\mu \cdot A}{l} \right)$$

The cross-sectional area of the magnetic path within the solenoid is about

$$A = \pi \cdot r^2$$

Outside the solenoid, the field is also in air and is assumed to have a similar path cross-section. The resulting solenoid inductance approximation is thus

$$L_{solenoid} \approx N^2 \cdot \mu \cdot \left(\frac{\pi \cdot r^2}{l} \right)$$

With this background, we are ready to begin development of electric-machine theory in the next article.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For further reading on motors and motor drives, see "A Practical Primer On Motor Drives".