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Non-Inverting Integrators Are Not Really Integrators (Part 3): Impact On Rogowski Sensors

by Gregory Mirsky, Design Engineer, Deer Park, Ill.

In previous parts[1,2] we considered general properties of inverting and non-inverting integrators and figured out that a non-inverting configuration is *not* an integrator at all. We have come to that conclusion by analyzing the amplitude-frequency characteristics of both configurations. However, we would also like to know the impact of these characteristics in a key integrator application—the Rogowski coil sensor.

To do this, we'll analyze the responses of both the inverting and non-inverting integrator configurations to the derivatives of rectangular pulses. We are using rectangular pulses because they have a wide spectrum and thus represent the worst case. And while we've cast doubt on the value of non-inverting integrator configurations, we'll still include them in our analysis because of the frequency with which they appear in a plethora of publications.

First, we will create an analytic expression for two (or more) rectangular pulses and then analyze how their derivatives are processed by inverting and non-inverting integrators. These differentiated pulses represent processing of rectangular pulses by the mutual inductance M of the current bus and Rogowski coil. Some of the material referred to in this discussion was previously given in my article on Rogowski sensor design.[3]

Expressions For Pulses And The Rogowski Coil Response

Denoting the rectangular waveform of the bus current to measure as $I_b(t)$, current pulse amplitude as I_{b0}, pulse repetition period as T_S and duty-cycle as D_C, we can compose an expression for a series of two pulses using the following expression, where n is the number of pulses and n = 1 corresponds to two pulses.

$n_{max} = 1$

$$I_b(t) = I_{b0} \cdot \left[\sum_{n=0}^{n_{max}} \left[\Phi(t - n \cdot T_S) - \Phi[t - T_S \cdot (n + D_C)]\right]\right] \qquad (1)$$

For convenience of analysis, we will be using the Laplace transform when usable.

By taking Laplace transform of equation (1), assuming $T_S > 1$ and $D_C > 0$, and collecting I_{b0}, we get

$$\left[-\frac{e^{-T_S \cdot s(D_C+1)} - e^{-T_S \cdot s} + e^{-D_C \cdot T_S \cdot s} - 1}{s}\right] \cdot I_{b0}$$

Therefore, the Laplace transform of the rectangular waveform of the bus current is

$$I_b(s) = \left[-\frac{e^{-T_S \cdot s(D_C+1)} - e^{-T_S \cdot s} + e^{-D_C \cdot T_S \cdot s} - 1}{s}\right] \cdot I_{b0} \qquad (2)$$

As described in reference [3] the transfer function of the Rogowski coil is

$$V_{coil}(t) = M \cdot \left(\frac{d}{dt} I_b(t)\right) \qquad (3)$$

Moving to the frequency domain, the Laplace transform of this expression is

$$V_{coil}(s) = s \cdot M \cdot I_b(s) \qquad (4)$$

We will disregard some negative signs and constants present in reference [3] to facilitate the analysis of interest since these constants do not affect the result.

Hence,

$$V_{coil}(s) = s \cdot M \cdot \left[-\frac{e^{-T_S \cdot s(D_C+1)} - e^{-T_S \cdot s} + e^{-D_C \cdot T_S \cdot s} - 1}{s} \cdot I_{b0} \right]$$

Simplifying we get

$$V_{coil}(s) = (e^{-T_S \cdot s} - e^{-D_C \cdot T_S \cdot s} - e^{-T_S \cdot s} \cdot e^{-D_C \cdot T_S \cdot s} + 1) \cdot I_{b0} \cdot M \qquad (5)$$

This Rogowski coil voltage represented by the Laplace transform, reflects the differentiated bus current, and is applied to an integrator that might be either an inverting or non-inverting arrangement. We will consider them one-by-one.

Inverting Integrator's Response To Rogowski Coil Voltage

The transfer function of an inverting integrator is

$$G_{inv}(s) = \frac{1}{s \cdot R_{int} \cdot C_{int}} \qquad (6)$$

Here R_{int} and C_{int} are essential components of the inverting integrator.

The inverting integrator output, therefore, is

$$V_{out\_inv}(s) = V_{coil}(s) \cdot G_{inv}(s) \qquad (7)$$

Plugging in (5) and (6) into (7), we obtain

$$V_{out\_inv}(s) = (e^{-T_S \cdot s} - e^{-D_C \cdot T_S \cdot s} - e^{-T_S \cdot s} \cdot e^{-D_C \cdot T_S \cdot s} + 1) \cdot I_{b0} \cdot M \cdot \frac{1}{s \cdot R_{int} \cdot C_{int}} \qquad (8)$$

To return to the time domain, we apply inverse Laplace transform to (8). Assuming s > 0 and $D_C > 0$, and simplifying, we get

$$-\frac{I_{b0} \cdot M \cdot (\Phi(T_S - t) - \Phi(T_S - t + D_C \cdot T_S) + \Phi(t - D_C \cdot T_S) - 1)}{C_{int} \cdot R_{int}}$$

Thus

$$V_{out\_inv}(t) = \left[-\frac{I_{b0} \cdot M \cdot (\Phi(T_S - t) - \Phi(T_S - t + D_C \cdot T_S) + \Phi(t - D_C \cdot T_S) - 1)}{C_{int} \cdot R_{int}} \right] \qquad (9)$$

Non-Inverting Integrator's Response To Rogowski Coil Voltage

The transfer function of a non-inverting integrator is

$$G_{noninv}(s) = 1 + \frac{1}{s \cdot R_{int} \cdot C_{int}} = \frac{C_{int} \cdot R_{int} \cdot s + 1}{C_{int} \cdot R_{int} \cdot s} \qquad (10)$$

The non-inverting integrator output, therefore, is

$$V_{out\_noninv}(s) = V_{coil}(s) \cdot G_{noninv}(s) \tag{11}$$

Plugging in (5) and (10) into (11), we obtain

$$V_{out\_noninv}(s) = \left(e^{-T_S \cdot s} - e^{-D_C \cdot T_S \cdot s} - e^{-T_S \cdot s} \cdot e^{-D_C \cdot T_S \cdot s} + 1\right) \cdot I_{b0} \cdot M \cdot \frac{C_{int} \cdot R_{int} \cdot s + 1}{C_{int} \cdot R_{int} \cdot s} \tag{12}$$

Going back to the time domain, by taking the inverse Laplace transform of (12), we get

$$\frac{I_{b0} \cdot M \cdot (\Phi(T_S - t) - \Phi(T_S - t + D_C \cdot T_S) + \Phi(t - D_C \cdot T_S) - C_{int} \cdot R_{int} \cdot \Delta(t) - C_{int} \cdot R_{int} \cdot \Delta(t - T_S) + C_{int} \cdot R_{int} \cdot \Delta(t - T_S - D_C \cdot T_S) + C_{int} \cdot R_{int} \cdot \Delta(t - D_C \cdot T_S) - 1)}{C_{int} \cdot R_{int}}$$

and

$$V_{out\_noninv}(t) = -\frac{I_{b0} \cdot M \cdot (\Phi(T_S - t) - \Phi(T_S - t + D_C \cdot T_S) + \Phi(t - D_C \cdot T_S) - C_{int} \cdot R_{int} \cdot \Delta(t) - C_{int} \cdot R_{int} \cdot \Delta(t - T_S) + C_{int} \cdot R_{int} \cdot \Delta(t - T_S - D_C \cdot T_S) + C_{int} \cdot R_{int} \cdot \Delta(t - D_C \cdot T_S) - 1)}{C_{int} \cdot R_{int}} \tag{13}$$

Comparing equation (13) for the non-inverting configuration to equation (9) for the inverting type shows a huge difference caused by the fact that a non-inverting integrator cannot serve as an integrator due to presence of the delta function denoted as Δ that stems from the differentiating of the coil voltage and its direct transfer by the non-inverting input of the integrator.

To illustrate, let's assume the following values for a bus current waveform and an inverting integrator circuit:

V_{01} = 1 V

T_S = 20 µs

D_C = 0.6

C_{int} = 100 nF

R_{int} = 10 kΩ

C_{diff} = 100 pF

R_{diff} = 10 kΩ

$C_{int} \bullet R_{int}$ = 1 x 10^{-3} s

$R_{diff} \bullet C_{diff}$ = 1 x 10^{-6} s

I_{b0} = 100 A

M = 0.330 µH

and plug them into the equations for the bus current sensed by the Rogowski coil ($I_b(t)$):

$$I_b(t) = I_{b0} \cdot \left[\sum_{n=0}^{n_{max}} \left[\Phi(t - n \cdot T_S) - \Phi[t - T_S \cdot (n + D_C)] \right] \right]$$

and the inverting integrator output (Vout_inv(t)):

$$V_{out\_inv}(t) = -\frac{I_{b0} \cdot M \cdot (\Phi(T_S - t) - \Phi(T_S - t + D_C \cdot T_S) + \Phi(t - D_C \cdot T_S) - 1)}{C_{int} \cdot R_{int}}$$

we get the waveforms plotted in Figs. 1 and 2.

Having plotted the output of the Rogowski coil as processed by an inverting integrator, we may be tempted to do the same with a non-inverting integrator. Recalling equation (13),

$$V_{out\_noninv}(t) = -\frac{I_{b0} \cdot M \cdot (\Phi(T_S - t) - \Phi(T_S - t + D_C \cdot T_S) + \Phi(t - D_C \cdot T_S) - C_{int} \cdot R_{int} \cdot \Delta(t) - C_{int} \cdot R_{int} \cdot \Delta(t - T_S) + C_{int} \cdot R_{int} \cdot \Delta(t - T_S - D_C \cdot T_S) + C_{int} \cdot R_{int} \cdot \Delta(t - D_C \cdot T_S) - 1)}{C_{int} \cdot R_{int}}$$

However, in this case, there is no output to plot, because the non-inverting integrator output waveform could not be restored since the inverse Laplace transform could not transform a delta function. The delta function describes abrupt surges that represent differentiated rectangles.

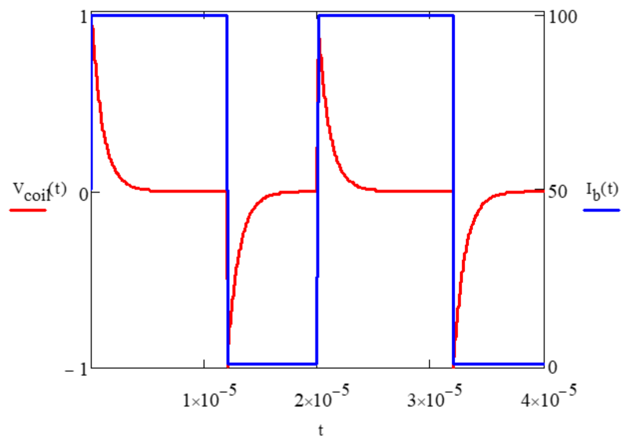

Fig. 1. Plot for the current to measure I_b (t) and output of the Rogowski coil V_{coil} (t), which is essentially a derivative of the current.

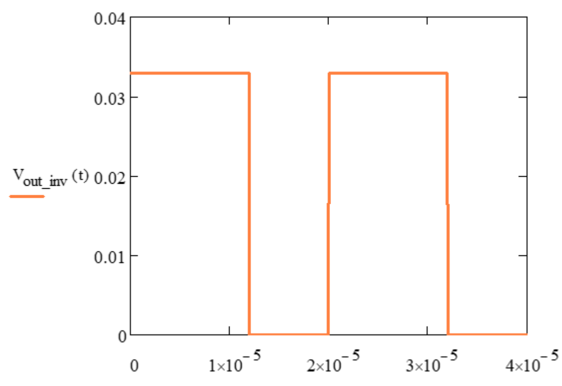

Fig. 2. Inverting integrator output is an accurately scaled replica of the current to measure I_b(t), derived from the Rogowski coil voltage.

Takeaway

Expression (13) for the non-inverting integrator, unlike expression (9) for the inverting integrator, has a differentiated component for the output voltage denoted with a red triangle (delta function) since it goes through the non-inverting part of the integrator, making its action null. The output of such a schematic has all the spikes coming from the Rogowski coil, and therefore, a non-inverting integrator is not viable.

References

1. "Non-Inverting Integrators Are Not Really Integrators (Part 1)" by Gregory Mirsky, How2Power Today, May 2025.

2. "Non-Inverting Integrators Are Not Really Integrators (Part 2)" by Gregory Mirsky, How2Power Today, June 2025.

3. "A Guide To Designing Your Own Rogowski Sensor (Part 1)" by Gregory Mirsky, How2Power Today, April 2024.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.