

Motor Control For Designers (Part 2): Electromagnetic Force Production In Motors

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As part 1 introduced basic terms and concepts of electricity and magnetism relevant to motors,* this part will continue the discussion of motor theory by explaining how electromagnetic force is generated. This article begins by introducing the Lorentz Force equation to explain the generation of mechanical force in electrostatic and magnetic motors.

Next, we use the example of a simple two-pole motor to explain how the motor components (stator, rotor, windings, magnets) cause magnetic fields to interact to produce force and motion. The relationship between rotor field and stator field-current vectors is used to explain phase and magnitude control as the basis for motion control including torque generation.

The next section extends the explanation of motor construction, describing an example of an outer-rotor permanent-magnet (PM) motor including pole-pairs, winding configurations, and various aspects of its operation. Finally, the roles of pole-pairs and phase windings in the implementation of phase control and the regulation of torque angle (relationship between stator and rotor field vectors) for field-oriented control are explained.

Relating Electric And Magnetic Fields To Mechanical Force

A motor converts electrical energy to mechanical energy through the medium of a field. The first method of relating E&M to mechanics is the Lorentz force equation;

$$\text{Lorentz Force Equation: } \mathbf{F} = q \cdot \mathbf{E} + q \cdot \mathbf{u} \times \mathbf{B}$$

This equation appeals to causal reasoning. For magnetic motors, it is the second term that applies, where \mathbf{u} is a velocity vector with magnitude u which is speed.

Field quantities can also be expressed as energy density as can mechanical energy, and equated as

$$\text{Energy Relation: } W = \int \mathbf{F} \bullet d\mathbf{x} = \frac{1}{2} \cdot \int_V \mathbf{B} \bullet \mathbf{H} \cdot dv$$

This second *energy method* results in a more general theory for electromechanical engineering.

We will first apply the Lorentz force equation:

$$\mathbf{F} = q \cdot \mathbf{E} + q \cdot \mathbf{u} \times \mathbf{B} = q \cdot \mathbf{E} + i \cdot \mathbf{I} \times \mathbf{B}$$

where

$q \cdot \mathbf{E} \Rightarrow$ electrostatic motors

$i \cdot \mathbf{I} \times \mathbf{B} \Rightarrow$ magnetic motors

For $i \cdot \mathbf{I}$ and \mathbf{B} at right angles, $F = i \cdot I \cdot B$; speed $u = dl/dt$ and current $i = dq/dt$. Then

$$\begin{aligned}
 q \cdot u &= q \cdot \frac{dl}{dt}, q \text{ constant} \\
 &= l \cdot \frac{dq}{dt}, l \text{ constant} \\
 &= i \cdot l
 \end{aligned}$$

where l is the winding length along which force is produced. (Motor end-turns are in-line with \mathbf{B} and do not produce force.) Either a mechanical change of length in time or speed u of a fixed l results in force production or else a change in charge q in time as current produces F . In either case, relative motion is occurring: $q \cdot u$ is fixed charge q , moving at a rate of u ; $i \cdot l$ is the rate of charge moving through fixed length l . We have the same predicament with motors as with magnetic components: nothing happens unless a field quantity is changing.

We will need to pay *some* attention to vector calculus in the application of the *right-hand rule* of cross-product in the Lorentz force equation:

$\times \Rightarrow$ cross product \Rightarrow right-hand rule: **I**: first finger; **B**: bent 2-4 fingers 90 degrees; resultant **F**: thumb

To show three dimensions on 2D screens or paper, we use the convention for vectors depicted as arrows:



tail of arrow: *into* plane of page



head of arrow: *out of* plane of page

To reduce the vector math to algebra, we work with quantities at right angles. This reduces cross product to scalar multiplication and to algebraic equations.

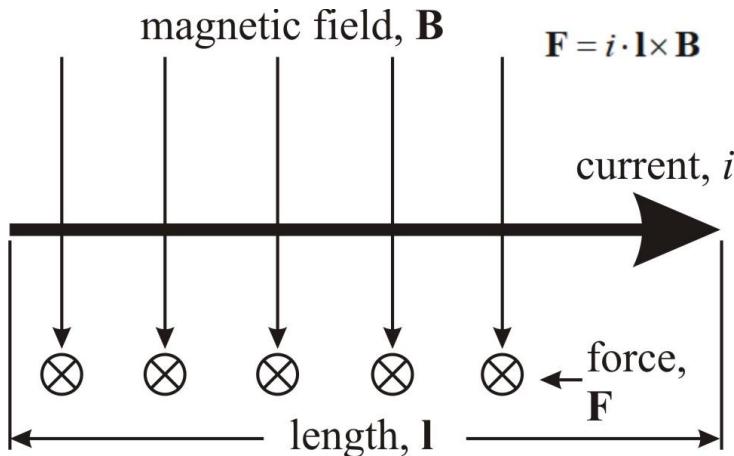


Fig. 1. Lorentz force equation in graphical form. Magnetic field \mathbf{B} direction is downward, cutting across current i along length l (as $i \cdot l$). Current flows from left to right, \mathbf{B} -field from top to bottom. This results in a force of magnitude F into the page along length l of the conductor of current i .

Simple Two-Pole Motor

The basic motor components are somewhat like those of a transformer, where two windings are coupled by a magnetic field in a core. What is different about motors is that the secondary "winding" is not electrical but is a

mechanical “moving winding,” though the coupling between winding and mechanics is also through a magnetic field.

The “core” is the *stator*, the stationary force-production component of the motor. The *rotor* is the rotational force-production component that for permanent-magnet (PM) synchronous (PMS) motors have PMs attached to it that create the rotor *B*-field. As the rotor turns, the PM flux cuts across the stator windings, producing force that turns the rotor. PMs of alternating *B* polarity are mounted around the rotor, and windings are placed in *windows* around armature *teeth* of the stator, as shown in Fig. 2.

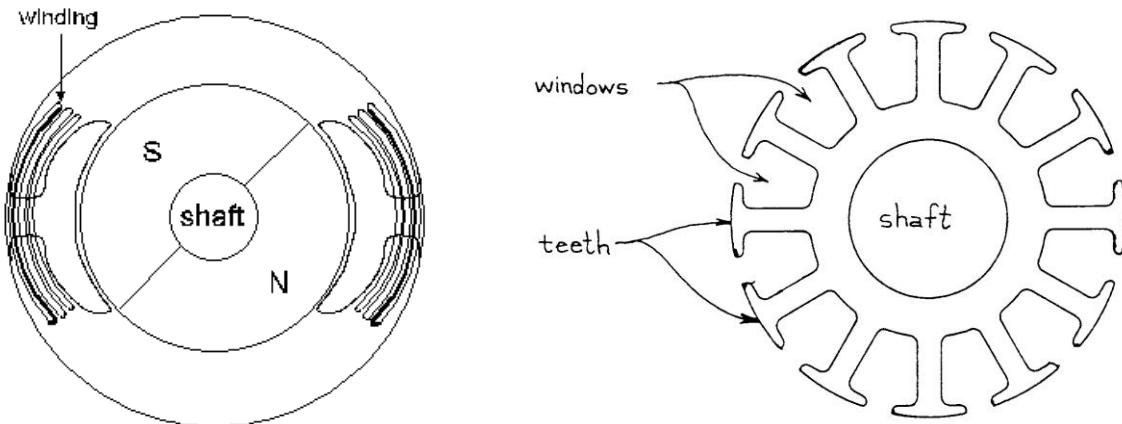


Fig. 2. On the left is a cross-sectional view of a simple PM inner-rotor motor with two windings on two stator teeth. The magnets are attached to the shaft and they turn (synchronously) with it. On the right is a view of an outer-rotating motor where a ring of magnets (not shown) rotate around a stator with 12 teeth.

The PMS motor of Fig. 2, left has an inner-rotating rotor and an outer stator. The alternative configuration is to exchange them geometrically so that the rotor goes around an inner stator, as shown on the right of Fig. 2 (and with the rotor shown in Fig. 6). Each has its relative merits.

On the left, the magnets are half-cylinder opposite-polarity magnets, typically made to include rare-earth elements. The most common magnet material is neodymium-iron-boron (NdFeB).

It is common for stator material to be electrical (3% Si) steel. If you are familiar with switching converter magnetics, you know that iron (steel) has high hysteresis loss that is excessive for switching frequencies above the audio range. However, motor dynamics usually have electrical frequencies in the audio range and electrical steel is optimal because of its high field saturation range.

For magnets, *magnetic polarity* follows the convention in vector calculus and is

B-field comes *out* of N pole surface of magnet (+)
B-field goes *into* S pole surface of magnet (-)

In the inner-rotor motor, current in the stator winding(s) produces a stator field that interacts with the PM rotor field. The angle between these two field vectors is the *torque angle* shown in Fig. 3. The goal of motor-drive *phase control* is to keep δ at a right angle of $\pi/2$ (90°). The magnitude of the stator field vector is controlled by the driver current magnitude and it controls the torque magnitude. Consequently, motor control is *vector control* in that magnitude and phase are both in need of control.

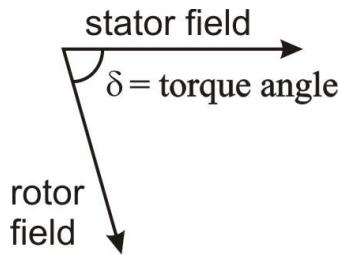


Fig. 3. The angle between field vectors is the torque angle δ . For $\delta = \pi/2$ (90°), the phasing between the two fields is field-oriented and maximum torque is produced in only the desired direction.

Stator electromagnets attract rotor magnets. The stator field vector stays ahead of the PM rotor field to pull rotor magnets along rotationally. Maximum torque T occurs whenever the stator field vector leads the rotor field vector by $\pi/2 = 90^\circ$. The stator field vector of a two-pole motor as in Fig. 2, left is 1D (horizontal only) and cannot rotate magnets (much) but only align them horizontally. (For some applications having external torque applied to the shaft, this is useful, with a range a fraction of a circle.) To create a rotating field-current vector, stator flux must be generated in both cross-sectional dimensions, as in Fig. 4.

Two-Pole Synchronous Motor

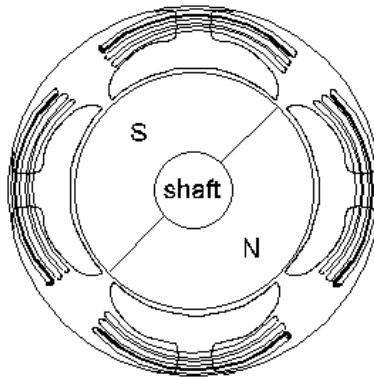


Fig. 4. A stator capable of creating magnetic flux in 2D, with both horizontal and vertical field current or flux production. The result is a rotating stator field vector with x and y winding current components.

The two separate winding-pairs, or *phase-windings*, are independent and can generate x - and y -axis stator fields. By controlling x (A) and y (B) winding currents, a stator 2D B -field vector is generated, and can be rotated in time when driven by the following current waveforms:

$$i_A = I \cdot \cos \theta$$

$$i_B = I \cdot \sin \theta$$

Then the stator field-current vector has constant magnitude with circuit current

$$\text{Magnitude} = \sqrt{i_A^2 + i_B^2} = I, \quad \angle = \theta$$

where θ is the angle of \mathbf{B} measured relative to the stator reference-frame x -axis. The rotor field vector maximum B orientation is called the *direct (d)* axis, positioned at the center of the magnet. The stator field vector produced from stator current leads the d -axis by $+90^\circ$ and is the torque-producing *quadrature (q)* axis.

Motor control is vector (phase and magnitude) control. Phase control maintains $\delta = 90^\circ$. Magnitude control adjusts amount of torque.

PMS Motor Construction

An outer-rotor PM configuration is shown in Fig. 5. Closed magnetic loops are formed from *pairs* of magnet poles. *Pole-pairs p* are a more convenient quantity in equations than poles; poles always are in pairs because magnetic paths are always closed-loop. What is not shown in Fig. 5 is that the paths are completed in closure on the back sides (outside) the magnets, through steel ("iron") holding the magnets, referred to as "back-iron".

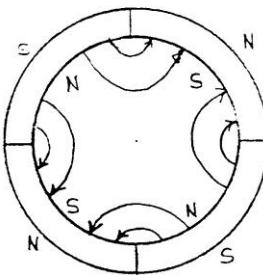


Fig. 5. PM placement for an outer-rotating rotor, with stator inside it with teeth extending outward, as in Fig. 2, right. Alternating N and S magnets on the rotor form pairs of poles, with fields as shown. With a stator, the field lines extend through the stator armature.

Though two phase-windings at minimum rotate a field vector in 2D, three phase-windings are the most common, with phases separated by 120° electrical. Typical step-motors are PMS motors with two phase-windings and "servo-motors" are three-phase because the number of power switches is reduced from 8 to 6 in a three-phase H-bridge.

In Fig. 6, a typical outer-rotor PMS motor has a winding spanning three teeth and is wound between three winding windows. Torque-producing segments of wire run into and out of the page. The vertical segments shown are *end-turns* that do not produce torque. The motor radius r is from the air gap side of the rotor magnets. The direction of \mathbf{B} produced by the windings is normal to and concentrated in the iron teeth of the stator.

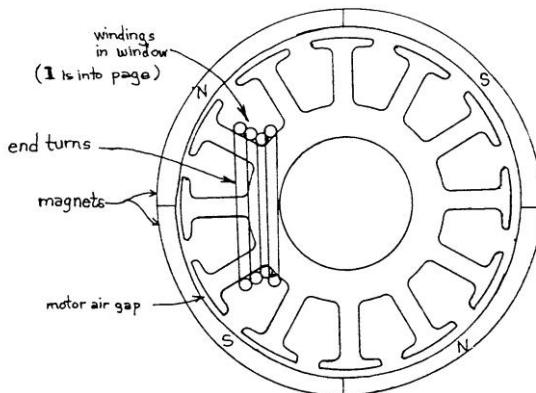


Fig. 6. End-turns of a winding spanning three stator teeth of an outer-rotor PMS motor.

To apply $\mathbf{F} = i\mathbf{I} \times \mathbf{B}$ in Fig. 7, the magnetic field \mathbf{B}_r of the rotor PM aligns with the high- μ stator teeth and is at right angles to the field produced by the current loop of the winding. Assume current flows into the page for the upper-window conductors of the winding. Use the right-hand rule: index finger points into page in direction of current; second (bent) finger points in direction of rotor \mathbf{B} (SE direction); thumb points in direction of force on

stator (SW) to move the stator in a CCW direction. The stator is stationary; the reaction force is on the rotor and it moves CW. That is, force is created between rotor and stator and one of them is free to move, and that is the rotor.

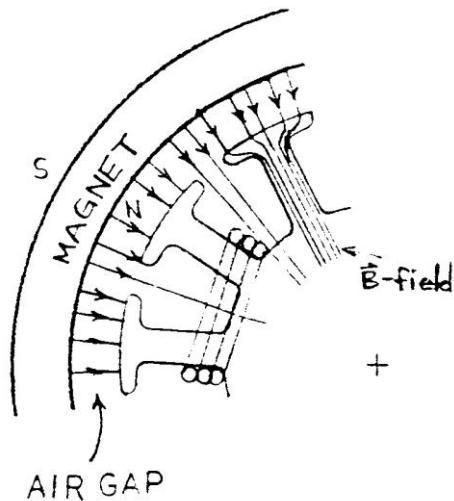


Fig. 7. Magnetic field from the N side of the rotating magnet is outward from the magnet into the adjacent stator teeth. The B-field is concentrated in the teeth. Torque produced by upper and lower turns of winding cancel.

Now consider the current in conductors in the lower winding window of Fig. 7 as out of the page. It produces force in a NE direction, whereas upper-window force production is in a SW direction, and they cancel! For net force generation, the rotor field must be of opposite magnetic polarity for upper and lower conductors. Thus, force is produced only when magnet edges sweep across winding loops. For Fig. 7, the magnet needs to be segmented so that an S pole field points inward into the magnet from the teeth of the lower window to produce torque in the same direction.

A full phase-winding (one of three) is shown in Fig. 8, consisting of four right-angle windings separated by 90° , alternately wound CW and CCW. With magnets of 90° arc (pole pitch = 90°), net torque is produced.

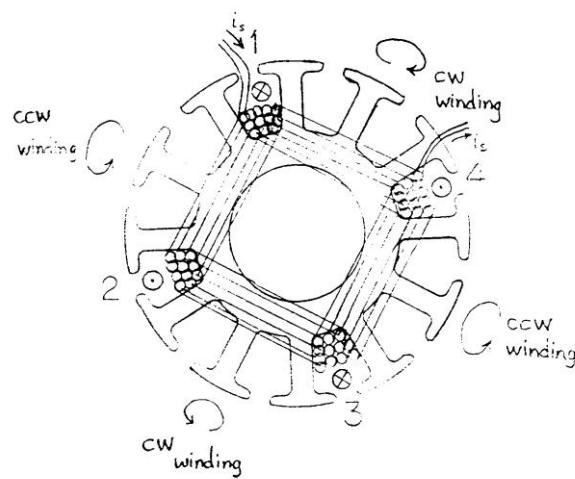


Fig. 8. Functional four-pole motor scheme, with alternating windings on stator and PM polarities on rotor.

Assume N magnet poles over windows 1 and 3 and S poles over windows 2 and 4. Generated force is between stator windings and magnets. The rotor will rotate in a direction opposite to stator torque, from the reaction torque. CCW force on stator conductors results in CW rotor rotation. The force production on stator (opposite on rotor) and current polarities are given in the table.

Table. Magnetic polarities and force production for motor of Fig. 8.

Conductors	Rotor Field	Conductor Current	Force Polarity
1	N	into page	CCW
2	S	out of page	CCW
3	N	into page	CCW
4	S	out of page	CCW

Poles-Pairs, Phase Control And Phases

Multiple pole-pairs distribute torque production around the axial (θ) dimension of the stator. To achieve phase control, phase-windings are driven with current of a polarity that produces torque in same direction. The stator field vector leads the rotor field vector by $\delta = 90^\circ$, resulting in *field-oriented control*. The stator field is synchronous with (but leading by 90°) the rotor magnet field. To implement this with circuits, rotor position is sensed to determine where the rotor field vector is pointing to *commutate* (switch-sequence) the stator phase-winding drive currents.

There is one magnetic cycle per two poles; each pole is one half-cycle. Stator current cycles follow poles: for p pole-pairs are p full electrical cycles per mechanical cycle. Electrical frequency \geq mechanical frequency:

$$\omega_{el} = p \cdot \omega_{me}$$

Thus for four poles there are two pole-pairs, and electrical frequency is twice the mechanical frequency, which is often measured in rev/m (rpm) but more conveniently for design in rev/s = Hz me.

A 3600-rpm rotor spins at a mechanical frequency of 60 Hz me. For a four-pole motor, the electrical frequency of the currents driving the windings is 120 Hz el. Following the Hz unit with "me" (mechanical) or "el" (electrical) indicates the reference-frame of the quantity.

Phase-windings each have the same number of pole-pairs and are spatially symmetrical along the axial axis. The motor in Fig. 9 has four poles of each of the phase-windings U, V, and W that are offset from each other by 360° me/12 teeth = 30° me tooth spacing.

The electrical phasing is $p \cdot 30^\circ$ me = 60° el. U loops are offset from W loops by 30° me = 60° el and from V loops by 60° me = 120° el. W is offset from U by 60° el, but is driven with opposite polarity current (180° el). Then W is separated in phase from U by 60° el - 180° el = -120° el = $+240^\circ$ el. The motor-drive controller generates the three waveforms separated symmetrically by 120° el.

To design with three phases of waveforms, the phases are made explicit on a stator vector diagram, shown in Fig. 10. The stator field is generated by the vector sum of phase-winding field contributions. Rotor and stator B -fields rotate in synchronism, in same direction. Hence these motors are called *synchronous* motors.

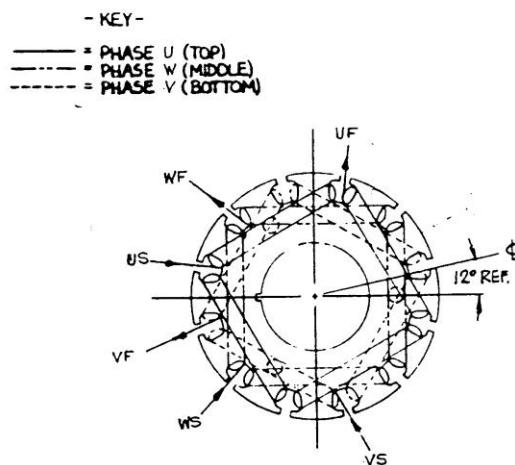


Fig. 9. Four-pole PMS motor with three phase-windings U, W, and V, with start (S) and finish (F) ends marked to indicate winding polarity. The phases are offset by one tooth pitch or $360^\circ/12 = 30^\circ$ me.

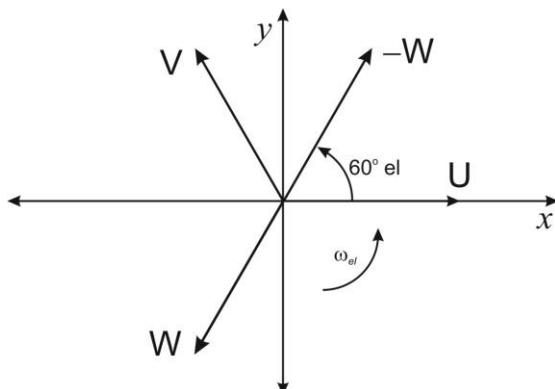


Fig. 10. Vector phase diagram showing explicitly the phase relationships between stator field-current vectors U, V, and W as they rotate CCW in the electrical frame. By inverting middle phase $-W$, electrical phase separation among phase-windings is 120° el.

The three phase-windings can be connected to each other in two different configurations: Y or Δ (delta), a topic for a subsequent article on motor theory in this series.

*Reference

["Motor Control For Designers \(Part 1\): Basic Principles Of Motor Theory"](#) by Dennis Feucht, Innovatia Laboratories, How2Power Today, July 2025.

About The Author



Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For further reading on motors and motor drives, see "[A Practical Primer On Motor Drives](#)".