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Motor Control For Designers (Part 3): Torque-Current Relationship

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

With the definition of terms and key relationships established in the first parts of this series,[1,2] we can now proceed to develop a motor model in the next three parts. As an electromechanical device, it will have certain relationships between electrical and mechanical quantities. This part relates electrical and mechanical quantities in two simple equations for torque T and induced voltage v_ω .

We begin by deriving these expressions, and then describe how motor geometry influences the induced-voltage waveform. This relationship is further illustrated by presenting the flux and induced-voltage waveforms for a simple PMS motor example.

Relating Torque To Stator Current

Torque is defined as a vector,

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

For **F** at right angles to the stator radius **r**, the cross-product torque magnitude is $T = r \cdot F \cdot \sin\theta$ and for $\theta = \pi/2 = 90°$, $T = r \cdot F$. Force and current are related by the Lorentz force equation from part 1:

$$\mathbf{F} = i \cdot \mathbf{l} \times \mathbf{B}$$

where $i \cdot \mathbf{l}$ and rotor magnet $\mathbf{B}_r$ are at right angles because the loops of the stator windings of length l produce a field that goes through the loops at right angles to the closed current path. Consequently, the force-current relation simplifies to

$F = i \cdot l \cdot B_r$

Conductors producing torque on opposite sides of the winding loop comprise a pole-pair equivalent to a single loop of width $2 \cdot r$, the stator diameter. Substituting into the torque equation and relating it to the stator current i_s, torque produced by a single pole-pair is

$$T = 2 \cdot r \cdot (i_s \cdot l \cdot B_r)$$

Stator windings have N turns per pole-pair. With p pole-pairs, stator terminal current i_s refers to the field by $p \cdot N$;

$$T = (2 \cdot r \cdot l) \cdot B_r \cdot [(p \cdot N) \cdot i_s] = \phi_r \cdot [(p \cdot N) \cdot i_s]$$

The rotor flux area A through the stator has flux $\phi_r = (2 \cdot r \cdot l) \cdot B_r = B_r \cdot A$. It interacts with field current $p \cdot N \cdot i_s$. To include phase-windings, i_s is the magnitude of the $\mathbf{i}_s$ vector (amplitude and phase) sum of currents in the phase-windings contributing to the $\mathbf{B}_s$ field vector. The final expression relating torque to stator current is thus

$$T = \left(p \cdot N \cdot (2 \cdot r \cdot l) \cdot B_r\right) \cdot i_s = \lambda_{me} \cdot i_s$$

where p = pole-pairs, l = torque-producing conductor length (approximately the stator core, or *armature*, length) on each side of the loop at a pole; B_r = average magnitude of the magnetic-field density of the magnets, and $2 \cdot r \cdot l$ is the area of rotor flux through the stator windings;

$$\lambda_{me} = p \cdot N \cdot (2 \cdot r \cdot l) \cdot B_r = p \cdot N \cdot A \cdot B_r = p \cdot N \cdot \phi_r$$

The quantity λ_{me} is the *mechanically-referred circuit flux*, the quantity that relates electrical and mechanical sides of the motor as the el ⇔ me conversion parameter. In the equation $T = \lambda_{me} \cdot i_s$, we have our first of two important equations, the one that relates mechanical torque to electrical stator current magnitude. It shows that torque magnitude is proportional to the magnitude of the stator current vector.

A nuanced side-note: the **B** in the Lorentz force equation is not $\mathbf{B}_r$ alone but is the total **B** combination of $\mathbf{B}_r$ and $\mathbf{B}_s$. Usually, the rotor field dominates, but at high torque—that is, at high stator current $\mathbf{i}_s$—the effect of the stator field becomes significant and causes a shift in phase of the total field, an effect called *armature reaction* in motor textbooks. The dominant $\mathbf{B}_r$ field will continue to be used here as such, but keep armature reaction in the back of your mind for more-refined treatment of phase control.

Speed-Induced-Voltage Relationship

Magnets moving across windings induce a voltage into them according to Faraday's Law (ignoring polarity);

$$v = \frac{d\lambda}{dt}$$

An S to N magnet edge change causes ΔB from $-B$ to $+B$, or $\Delta B = 2 \cdot B$. The change in (field-referred) flux ϕ is $\Delta\phi = 2 \cdot B \cdot A$. The *induced voltage* in the stator windings caused by the motion of the magnets across them is thus

$$v_\omega = \frac{d\lambda}{dt} = \frac{d(p \cdot N \cdot \phi)}{dt} = p \cdot N \cdot \Delta B \cdot \frac{dA}{dt} = p \cdot N \cdot (2 \cdot B) \cdot \frac{dA}{dt}$$

The rate of magnet area change *dA/dt* depends upon rotor speed;

$$u = (2 \cdot \pi \cdot f_{me}) \cdot r = \omega_{me} \cdot r$$

The symbol *u* is used for the speed of the rotor magnets at radius *r* (at the air gap between rotor magnets and stator teeth) because *v* already designates voltage, and both variables appear in motor equations. $\mathbf{B}_r$ moves at right angles to torque-producing conductors of length **l** so that

$$\frac{dA}{dt} = l \cdot u = l \cdot \omega_{me} \cdot r$$

Substituting *dA*/*dt*, the induced voltage is related to the mechanical speed by

$$v_\omega = (p \cdot N \cdot 2 \cdot r \cdot l \cdot B) \cdot \omega_{me} = \lambda_{me} \cdot \omega_{me}$$

The *induced voltage* v_ω (or "speed voltage") is related to mechanical speed by the same conversion factor λ_{me} as torque is to stator current. Thus, our two el ⇔ me conversion equations relating the two "sides" of the motor are

$$T = \lambda_{me} \cdot i_s$$

$$v_\omega = \lambda_{me} \cdot \omega_{me}$$

$$\lambda_{me} = p \cdot N \cdot (2 \cdot r \cdot l) \cdot B_r = p \cdot N \cdot A \cdot B_r = p \cdot N \cdot \phi_r$$

These conversion equations are the basis for developing a motor model. For now, we look in more detail at the waveforms that result from magnets moving across windings.

Induced-Voltage Waveform From Motor Geometry

As a magnet edge sweeps across the face of a winding, a rotor flux change penetrates the winding loop area, and as it changes with motion, the magnet field B_r induces voltage v_ω into the winding in proportion to the rate of flux change. Constant ω_{me} and B_r result in linearly changing circuit-referred winding flux $\lambda(t)$. Induced voltage is the time derivative of λ or $v_\omega = d\lambda/dt$ as shown in Fig. 1.

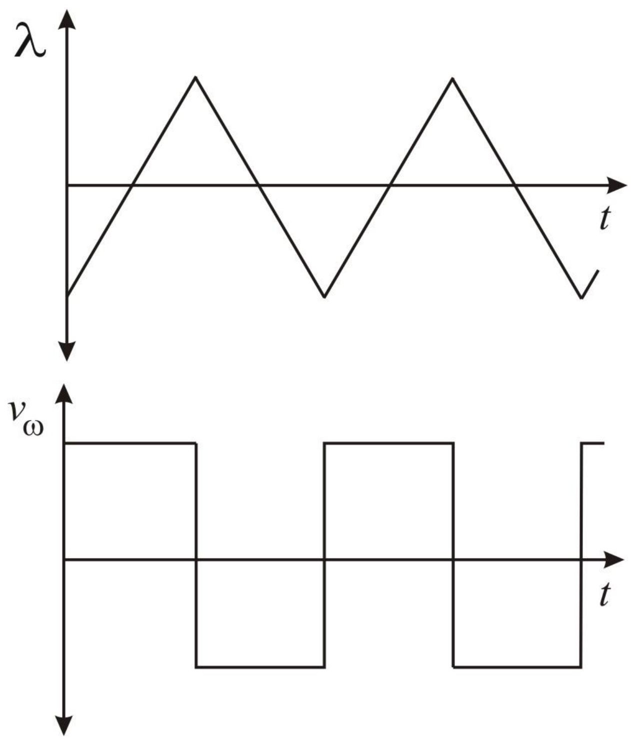

Fig. 1. A change in winding-referred circuit flux λ caused by the linear sweep of area by rotor magnets induces into the winding being swept a voltage v_ω that is the time derivative of the flux.

The waveform shown in Fig. 1 is produced with *full-pitch* poles; magnet arc length = winding arc length. For magnet arcs shorter than the winding arc lengths, no voltage is induced while $\Delta\lambda = 0$ during which the two magnet edges are within the loop area. For a given winding loop, all turns go through same armature windows (or *slots*), resulting in a square wave of induced voltage (Fig. 2). If turns are distributed spatially in the slots across multiple teeth, other waveforms result (such as sinusoids).

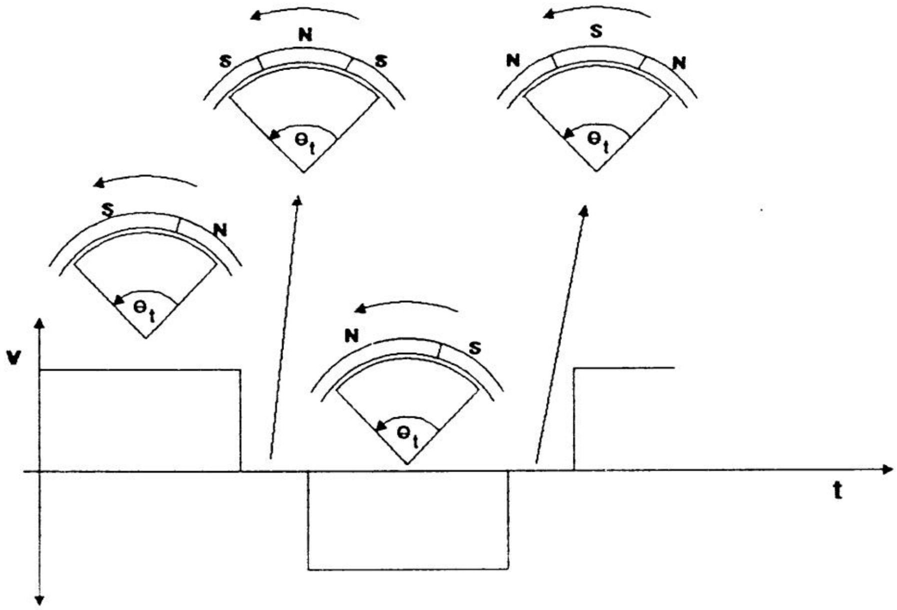

Fig. 2. A given winding has area exposure over an arc of angle θ_t. As rotor magnet edges pass over this arc, the flux presented to the winding changes. The magnet arcs are shorter than the stator winding loops so that when two edges are in the winding arc, $\Delta\lambda$ = 0 and the induced voltage is zero.

Example Of Induced Voltage Waveforms For A Simple Motor

As an example, a Mabuchi RE-36 motor, shown in Fig. 3, is a low-cost motor with windings phase-controlled by brushes and commutator bars electromechanically as a "dc brush" motor. The electromechanics have been removed and motor altered so that in Fig. 3 we are looking at a "brushless" motor (with front end-bearing removed from the shaft for the photograph), exposing three stator teeth, each with a winding. The two rotor magnets are blue (BLU) or S (–) and white (WHT) or N (+). The motor is converted to a PMS motor by making the inner rotor the stator (by holding the shaft motionless) and allowing the magnets and case (back-iron) to be the outer rotor.

Fig. 3. A Mabuchi RE-36 motor with end-bearing removed (for photo). The windings are brought out the other end and the stator and shaft made stationary while the magnets and case, comprise an outer rotor. (Movement range is less than one mechanical revolution because of the wire access window in the rear-bearing plate.)

To simplify the rotational machine for analysis, the circular structure is flattened to be linear as a *developed diagram* as shown in Fig. 4. For this motor, the graph has 24°/div, the motor is two-pole with three phase-windings, U, V, and W and the two magnets WHT and BLU.

To simulate rotor motion, shift the magnets to the left over the windings one division per iteration, and draw flux waveform segments. The flux magnitude is the amount of magnet-winding overlap of a given magnetic polarity. In the second graph down, phase-windings are connected to subtract flux in pairs: XY = X – Y. (Later, we will find that this is a Y winding configuration.)

Induced voltages in series-pairs of phase-windings are time derivatives of series-pair fluxes. Voltage waveforms are square-waves with dead-times between half-cycles. The motor geometry is not full-pitch; winding lengths do not equal magnet lengths—hence the 30°/div scale resolution.

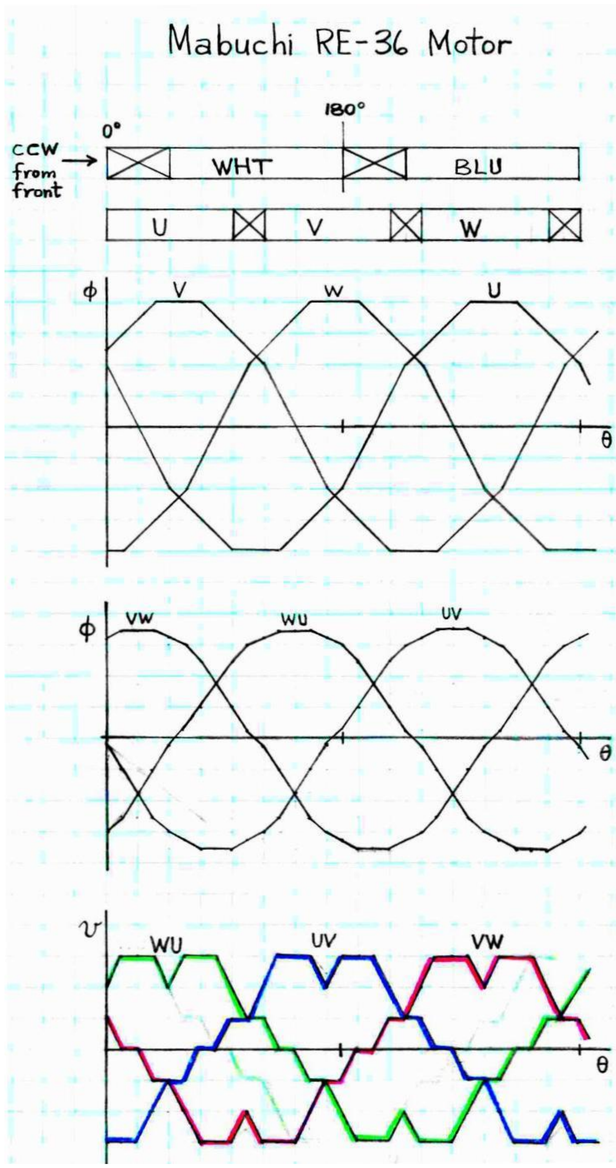

Fig. 4. Developed diagram of modified Mabuchi RE-36 motor, showing phase-winding fluxes, phase-winding flux differences for series-connected winding pairs, and induced-voltage waveforms of winding pairs in series.

At $\theta = 0$, U and V have positive flux values because the WHT magnet (+ flux) is over them. Winding W is completely covered by BLU and has a maximum negative flux value. Slide the rotor farther to the left, and whenever an edge appears over a winding, a change in flux for it begins. This is how the $\phi(\theta)$ function is graphed. Then subtract in pairs the phase-winding fluxes to result in the next lower $\phi(\theta)$. The windings are connected external to the motor with XY = X − Y polarities. Take the derivatives of the winding-pair fluxes to produce the lowest graph of the induced voltages, color-coded by phase-winding pair.

Typical induced-voltage waveforms are trapezoidal for motors wound full-pitch as in part 2, Fig. 9 (with all turns of a winding traversing the same tooth windows). In smaller motors (typical of hard-disk-drive motors) waveforms are "peaked sine-waves" because the magnet and winding segments are so short that flux fringing is appreciable. And some servomotors are intentionally sine-wave wound by varying turns among teeth. The waveshape also varies depending on how the phase-windings are interconnected.

We will look at induced-voltage waveshape more in the second series, on motor design. By knowing something about how motors are designed, the motor-drive designer can better optimize electronics with the electric machine in the same way that a power-circuits designer can optimize converter circuits by knowing something about magnetics design. Before that, our next motor theory articles develop a motor circuit model.

References

1. "Motor Control For Designers (Part 1): Basic Principles Of Motor Theory" by Dennis Feucht, Innovatia Laboratories, How2Power Today, July 2025.

2. "Motor Control For Designers (Part 2): Electromagnetic Force Production In Motors" by Dennis Feucht, Innovatia Laboratories, How2Power Today, August 2025.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For further reading on motors and motor drives, see "A Practical Primer On Motor Drives".