Exclusive Technology Feature

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Controlling Rogowski Sensor Frequency Response Through Integrator Design

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In a previous article series,[1,2,3] the operation of the Rogowski coil was analyzed and equations were derived for designing both the coil and integrator to create a complete Rogowski sensor. The purpose in doing so was to allow designers to obtain some of the benefits of Rogowski sensors such as wide current range at a much lower cost than off-the-shelf instrument-grade Rogowski sensors, while only making small sacrifices in sensor accuracy.

In this article, we carry the discussion design further by explaining how to tailor the frequency bandwidth of the Rogowski sensor to a particular application through adjustment of the integrator design. This part draws on some of the analysis performed in the recent series on integrators.[4] This article begins by repeating some of the material previously presented in part 1 of the Rogowski sensor design series[1] in deriving the Rogowski coil transfer function. But then the math changes as we move toward the design of the integrator, which represents a continuation of the discussion in another article series.[4,5,6]

Reviewing Rogowski Sensor Basics

First, let us set up the terminology: The Rogowski probe is the complete instrument composed of the coil and integrator and designated to operate as a standalone instrument. The Rogowski sensor is the complete instrument, similar to the probe, intended to work in a system as a part of the system continuously. The Rogowski coil is just the wire-wound coil around a dielectric flexible core, not connected to any integrator.

Rogowski ac current probes (sensors) are very accurate, versatile and convenient instruments that can be used for measuring ac currents over a very wide range of values and frequencies. Looked at another way, they are simply isolated wideband transimpedance amplifiers. They may be used as part of current measuring instruments and as standalone current sensors operating as a part of a control loop.

Rogowski sensors are based on a non-magnetic current measuring method, which does not employ any magnetic materials and therefore these sensors never saturate. They have a disconnectable sensing coil that can be wrapped around the current-carrying bus and locked to itself providing access to wires, pins and buses concealed in the depth of electronic and electric equipment. The sensing coil is terminated by an integrator (Fig. 1).

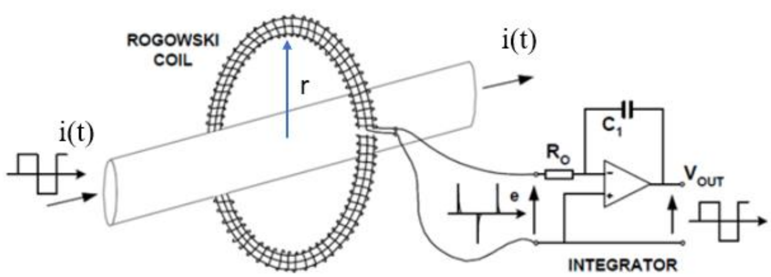

Fig. 1. Schematic of the Rogowski probe. The Rogowski coil wires are connected to an integrator that compensates for the differentiating property of the mutual inductance between the bus carrying current i(t) and the coil placed at a distance r from the bus. Diagram courtesy of Keysight (r and i(t) added here) (see reference [7]).

Rogowski Coil Frequency Response

Let's consider Fig. 2 depicting the Rogowski coil parameters. From this figure, we'll derive the transfer function of the Rogowski coil.

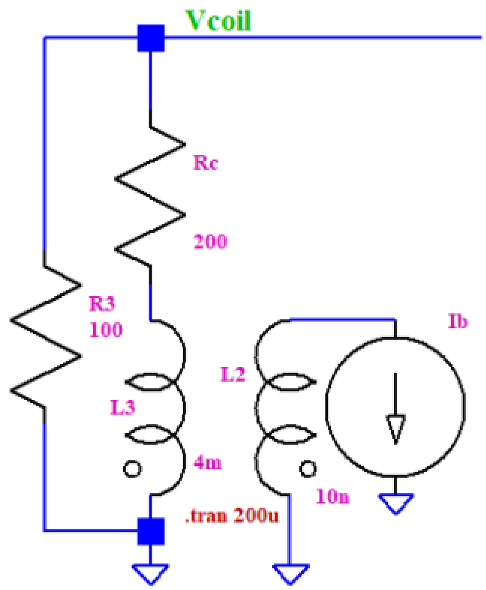

Fig. 2. This schematic diagram of a Rogowski coil also has a part, showing the bus represented by inductance L2 and current Ib to measure. Coupling between bus L2 and coil L3 is weak since there is a big air gap between them. Rc is the coil intrinsic resistance and R3 is a low value terminating resistor, which reduces noise and stabilizes the output Vcoil shape.

For convenience, let's list some of the crucial components of further equations.

$I_b(t)$ = the bus current to measure

N_b = number of the current-carrying bus turns

l_c = length of the Rogowski coil

μ_r = relative permeability of the coil core. In our case $\mu_r = 1$.

μ_0 = absolute permeability of free space

$\mu = \mu_0 \cdot \mu_r$

H(t) = magnetic field strength created by current in the bus $I_b(t)$

B(t) = magnetic flux density that crosses the coil turns

ψ(t) = magnetic flux linkage that crosses the coil turns

S_c = coil cross-sectional area

L2 = inductance of the bus (not needed for subsequent calculations)

L3 = inductance of the coil formed by the winding

R_c = intrinsic resistance of the Rogowski coil L3

M = mutual inductance between L2 and L3

R3 = termination resistance of the Rogowski coil

V_{coil} = voltage between the wires of the Rogowski sensor coil

V_{L3} = voltage across L3 induced by the magnetic field produced by the bus.

Using Ampere's law (Full Current Law) we can define the magnetic flux linkage in the coil:

Ampere's law:

$$H(t) \cdot l_c = I_b(t) \cdot N_b \qquad (1)$$

Hence the magnetic field crossing the coil turns is

$$H(t) = \frac{I_b(t) \cdot N_b}{l_c} \qquad (2)$$

Therefore, flux density

$$B(t) = H(t) \cdot \mu = \mu \cdot \frac{I_b(t) \cdot N_b}{l_c} \qquad (3)$$

and linkage flux

$$\psi(t) = S_c \cdot N_w \cdot B(t) = S_c \cdot N_w \cdot \left(\mu \cdot \frac{I_b(t) \cdot N_b}{l_c}\right) \qquad (4)$$

Designating the mutual inductance as M, we obtain the voltage generated across the wires of the Rogowski coil regardless of R_c as

$$V_{L3}(t) = \left(\frac{d}{dt}\psi(t)\right) = S_c \cdot N_w \cdot N_b \cdot \frac{\mu}{l_c} \cdot \left(\frac{d}{dt} I_b(t)\right) = M \cdot \frac{d}{dt} I_b(t) \qquad (5)$$

Applying the Laplace transform, we get

$$V_{L3}(t) = s \cdot M \cdot I_b(s) \qquad (6)$$

which yields the transfer function that has a dimension of inductance M in the time domain.

$$M = S_c \cdot N_w \cdot N_b \cdot \frac{\mu}{l_c}$$

We define the Rogowski coil transfer function from (6) as

$$R_{sen}(s) = \frac{V_{L3}(s)}{I_b(s)} = s \cdot M = s \cdot S_c \cdot N_w \cdot N_b \cdot \frac{\mu}{l_c} \qquad (7)$$

which is the coil sensitivity.

How are $V_{L3}(s)$ and $V_{coil}(s)$ related?

From Fig. 2 we see:

$$V_{coil}(s) = \frac{V_{L3}(s)}{s \cdot L3 + R_c + R3} \cdot R3 = V_{L3}(s) \cdot \frac{R3}{(R_c + R3) \cdot \left(1 + s \cdot \frac{L3}{R_c + R3}\right)} \qquad (8)$$

Designating

$$\sigma_0 = \frac{R3}{R3 + R_c} \qquad (9)$$

and

$$\tau_0 = \frac{L3}{R3 + R_c} \qquad (10)$$

we can transform (8) into

$$V_{coil}(s) = V_{L3}(s) \cdot \frac{\sigma_0}{1 + s \cdot \tau_0} \qquad (11)$$

Plugging in (6) and (11) into (7) we get

$$R_{coil}(s) = \frac{V_{coil}(s)}{I_{b0}(s)} = \frac{\frac{\sigma_0}{1 + s \cdot \tau_0} \cdot (M \cdot s \cdot I_{b0}(s))}{I_{b0}(s)} = \frac{\sigma_0}{1 + s \cdot \tau_0} \cdot s \cdot M \qquad (12)$$

Switching to the complex notation, we get

$$R_{coil}(s) = \frac{\sigma_0}{1 + j \cdot \omega \cdot \tau_0} \cdot j \cdot \omega \cdot M = \sigma_0 \cdot M \cdot \frac{j \cdot \omega}{1 + j \cdot \omega \cdot \tau_0} \qquad (13)$$

Assuming that

$$\frac{j \cdot \omega}{1 + j \cdot \omega \cdot \tau_0} = \frac{j \cdot \omega \cdot (1 - j \cdot \omega \cdot \tau_0)}{1 + (\omega \cdot \tau_0)^2} = \frac{\omega^2 \cdot \tau_0 + j \cdot \omega}{1 + (\omega \cdot \tau_0)^2}$$

we can obtain the frequency characteristic from (13):

$$R_{coil}(\omega) = \frac{\sqrt{(\omega^2 \cdot \tau_0)^2 + \omega^2}}{[1 + (\omega \cdot \tau_0)^2]} \cdot \sigma_0 \cdot M$$

where dimension is Ω.

That is,

$$R_{coil}(\omega) = \frac{\sqrt{\omega^4 \cdot {\tau_0}^2 + \omega^2}}{\omega^2 \cdot {\tau_0}^2 + 1} \cdot \sigma_0 \cdot M = \left(\frac{\omega}{\sqrt{\omega^2 \cdot {\tau_0}^2 + 1}}\right) \cdot \sigma_0 \cdot M \qquad [\Omega] \qquad (14)$$

Now, referring to Fig. 3, we define the inverting integrator transfer function $G_{int}(s)$:

$$G_{int}(s) = \frac{Z_{FB}(s)}{R1} \qquad (15)$$

where $Z_{FB}(s)$ is the impedance of the feedback circuit composed of C1 and R8.

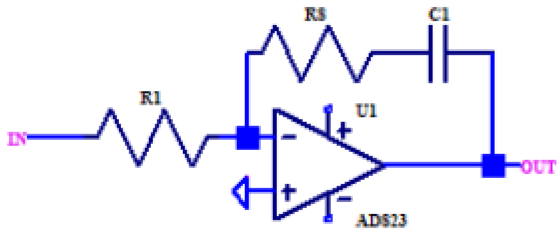

Fig. 3. Schematic diagram of a typical integrator without a dc feedback resistor that would perform as a servo feedback, removing the integrator dc walkaway.

$$Z_{FB}(s) = R8 + \frac{1}{s \cdot C1}$$

Simplifying and factoring results in

$$Z_{FB}(s) = \frac{C1 \cdot R8 \cdot s + 1}{C1 \cdot s} \quad (16)$$

The inverting integrator gain is therefore

$$G_{int}(s) = \frac{\frac{C1 \cdot R8 \cdot s + 1}{C1 \cdot s}}{R1}$$

or

$$G_{int}(s) = \frac{C1 \cdot R8 \cdot s + 1}{C1 \cdot R1 \cdot s} \quad (17)$$

The overall transfer function $W_0(s)$ can be obtained by multiplying (12) by (17):

$$W_0(s) = \frac{\sigma_0}{1 + s \cdot \tau_0} \cdot s \cdot M \cdot \frac{C1 \cdot R8 \cdot s + 1}{C1 \cdot R1 \cdot s} \quad (18)$$

Now, the integrator frequency response is

$$G_{int}(j \cdot \omega) = \frac{C1 \cdot R8 \cdot j \cdot \omega + 1}{C1 \cdot R1 \cdot j \cdot \omega} = \frac{(R1 \cdot R8 \cdot C1^2 \cdot \omega^2) - R1 \cdot C1 \cdot \omega \cdot j}{(C1 \cdot R1 \cdot \omega)^2}$$

$$G_{int}(\omega) = \frac{\sqrt{(R1 \cdot R8 \cdot C1^2 \cdot \omega^2)^2 + (R1 \cdot C1 \cdot \omega)^2}}{(C1 \cdot R1 \cdot \omega)^2}$$

Simplifying and factoring results in

$$G_{int}(\omega) = \frac{\sqrt{C1^4 \cdot R1^2 \cdot R8^2 \cdot \omega^4 + C1^2 \cdot R1^2 \cdot \omega^2}}{C1^2 \cdot R1^2 \cdot \omega^2}$$

This gives the modulus of the integrator frequency response i.e. frequency characteristic if simplified:

$$G_{int}(\omega) = \frac{\sqrt{C1^2 \cdot R8^2 \cdot \omega^2 + 1}}{C1 \cdot R1 \cdot \omega} \quad (19)$$

For simplicity we can multiply moduli of the frequency responses, (14) and (19):

The Rogowski frequency characteristic is copied from (14):

$$R_{coil}(\omega) = \frac{\sqrt{\omega^4 \cdot \tau_0{}^2 + \omega^2}}{\omega^2 \cdot \tau_0{}^2 + 1} \cdot \sigma_0 \cdot M = \left(\frac{\omega}{\sqrt{\omega^2 \cdot \tau_0{}^2 + 1}}\right) \cdot \sigma_0 \cdot M$$

and (18) becomes

$$W_0(\omega) = R_{coil}(\omega) \cdot G_{int}(\omega)$$

or

$$W_0(\omega) = \left(\frac{\omega}{\sqrt{\omega^2 \cdot \tau_0{}^2 + 1}}\right) \cdot \sigma_0 \cdot M \cdot \frac{\sqrt{C1^2 \cdot R8^2 \cdot \omega^2 + 1}}{C1 \cdot R1} \qquad (20)$$

Now, we can show an example of calculating parameters of a real Rogowski sensor. To begin, let's assume the following Rogowski coil component parameters:

$N_b = 1$

$\mu = \mu_0$

$l_c = 50$ mm

R3 = 100 Ω

Wire diameter: $d_W = 0.1$ mm

Number of turns: $N_w = \frac{l_c}{d_W} = 500$

One turn length: $d_{turn} = 20$ mm

Coil wire one-turn length: $l_{turn} = 50\ mm$

Coil wire length: $l_W = l_{turn} \cdot N_w = 25\ \mathrm{m}$

Coil winding one turn area: $S_C = 18\ \mathrm{mm} \times 3.3\ \mathrm{mm} = 5.94 \times 10^{-5} \mathrm{m}^2$

Coil inductance: $L3 = N_W{}^2 \cdot \frac{S_C}{l_c} \cdot \mu_0 = 373.221 \times 10^{-6} \mathrm{H}$

Coil resistance:

$\rho_{CU} = 1.68 \times 10^{-8} \Omega \cdot \mathrm{m}$

$$R_C = \rho_{CU} \cdot \frac{l_W}{\pi \cdot \frac{d_W{}^2}{4}} = 53.476\ \Omega$$

R1 = 10.0 kΩ

C1 = 0.01 µF

R8 = 0.024 kΩ

I_{b0} = 500 A

Mutual inductance

$$M = S_C \cdot N_W \cdot N_b \cdot \frac{\mu}{l_c} = 746.442 \text{ x } 10^{-9}\text{H}$$

$$\sigma_0 = \frac{R3}{R3+R_c} = 0.652$$

$$\tau_0 = \frac{L3}{R3+R_c} = 2.432 \; x \; 10^{-6} \text{ s}$$

Recalling equation 14,

$$R_{coil}(\omega) = \left(\frac{\omega}{\sqrt{\omega^2 \cdot \tau_0{}^2 + 1}}\right) \cdot (\sigma_0 \cdot M)$$

we can plug in the last three values just given and plot the amplitude-frequency characteristic of the example Rogowski coil as shown in Fig. 4.

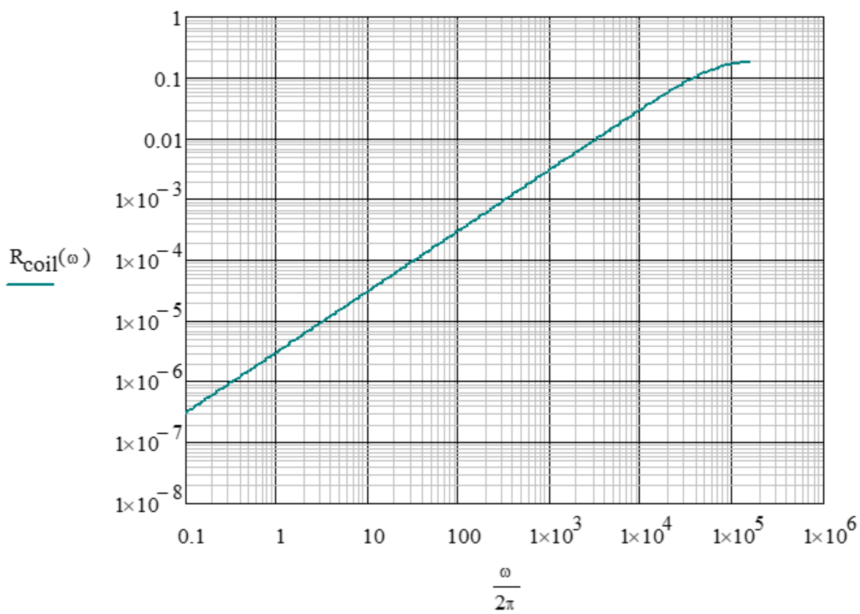

Fig. 4. Amplitude-frequency characteristic of the Rogowski coil. Its shape is that of a typical differentiator.

Then, recalling the integrator frequency response from equation (19),

$$G_{int}(\omega) = \frac{\sqrt{C1^2 \cdot R8^2 \cdot \omega^2 + 1}}{C1 \cdot R1 \cdot \omega}$$

we can plug in the example component values for the inverting integrator to obtain its amplitude- frequency characteristic as plotted in Fig. 5.

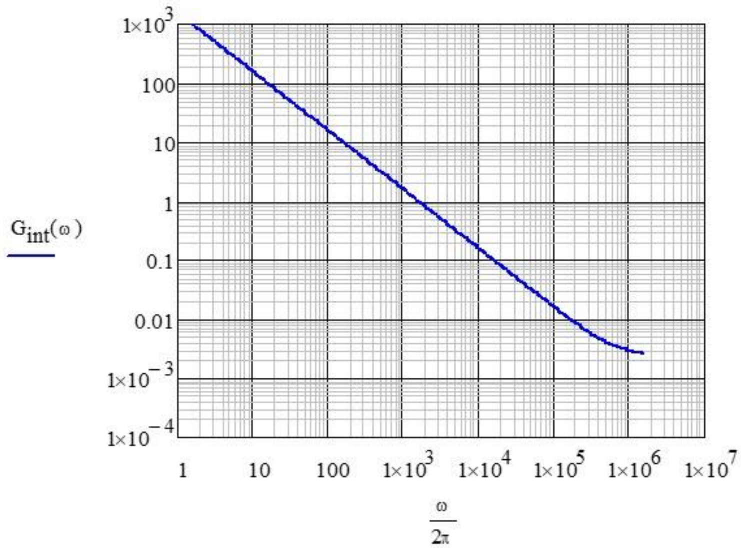

Fig. 5. A typical amplitude-frequency characteristic of an inverting integrator with a remote pole caused by the phase corrector C1-R8.

Recalling the modified version of equation (18) and equation (20), which give us the response of the complete Rogowski sensor,

$$W_0(\omega) = R_{coil}(\omega) \cdot G_{int}(\omega)$$

$$W_0(\omega) = \left(\frac{\omega}{\sqrt{\omega^2 \cdot \tau_0{}^2 + 1}}\right) \cdot \sigma_0 \cdot M \cdot \frac{\sqrt{C1^2 \cdot R8^2 \cdot \omega^2 + 1}}{C1 \cdot R1 \cdot \omega}$$

we can now use the example values to plot the inverting Rogowski sensor amplitude-frequency characteristic as shown in Fig. 6.

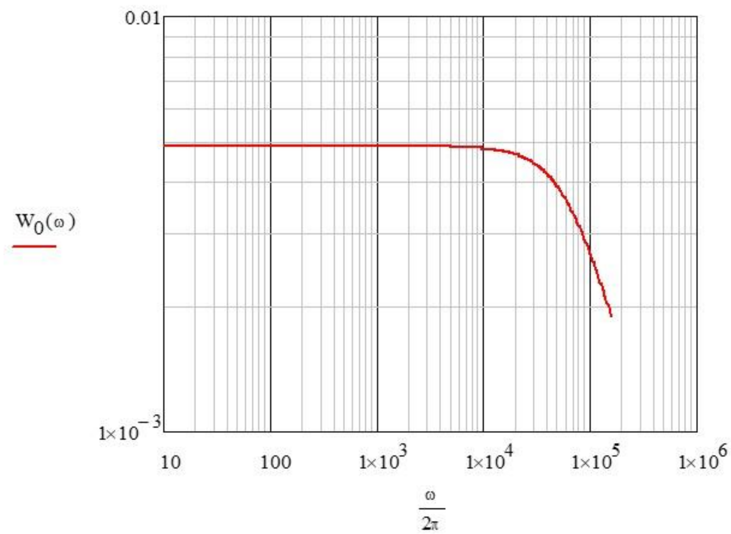

Fig. 6. Amplitude-frequency characteristic of a complete inverting Rogowski sensor. By tweaking the value of R8 one can adjust the integrator to better represent the measured current waveform, i.e. to adjust the operating frequency bandwidth.

Takeaway

An inverting integrator allows us to create wide-bandwidth Rogowski sensors, whose amplitude-frequency characteristic can be easily controlled by the components of the integrator. An extra roll-off of the characteristic around 1 Hz to 120 Hz for sensors operating in traction inverters is recommended to suppress low-frequency noise.

The schematics presented above explain how to compose a Rogowski sensor, but they cannot be used as shown. The reason for that is a significant "walkaway" of the integrator output due to thermal and noise effects at the amplifier's inputs and a lack of local dc feedback.

In future articles we will review what happens to the Rogowski sensor output when the integrator output and input are connected to a dc-stabilizing resistor, why it is unacceptable and how to work around it.

References

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About The Author

Gregory Mirsky worked as a design engineer in Deer Park, Ill. He performed design verification on various projects, designed and implemented new methods of electronic circuit analysis, and ran workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.