Why Couple The Windings In A SEPIC?

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The SEPIC (single-ended primary-inductor converter) is a nonisolated, switching power supply topology that provides an output voltage that can be higher, equal or lower than the input voltage. Its main applications are in power conversion systems with fluctuating input and/or output voltages, like automotive electronics, battery-based devices and chargers, offline LED lighting and power factor correction stages, among others.

The SEPIC power stage is built with two inductors, which could also share the same magnetic core as the two windings of a coupled inductor. This approach provides some advantages compared to an uncoupled design, like a lower ripple current amplitude on each winding, and it is common to hear that either the required inductance or the ripple current can be "halved" thanks to the magnetic coupling.

This article explains why the ripple current amplitude is reduced when magnetically coupling the windings in a SEPIC, as well as how the leakage inductance affects the maximum ripple current reduction. In addition, real measurements show one of the main advantages of using a coupled inductor in the SEPIC: higher converter efficiency for the same magnetics component size.

How Magnetic Coupling Reduces Ripple Current

In the SEPIC, the voltage across each inductor at any instant in time is the same (considering ideal components). It's this condition that enables the inductors' windings to share the same magnetic core.

However, when using one core for the two windings, it is important to observe the winding polarity for correct operation, which follows the same "dot" terminal convention as with transformers. Here, the dotted terminals must be both connected either to C_{AC} (i.e. to the switching nodes as in Fig. 1, recommended), or alternatively, to the two dc nodes.

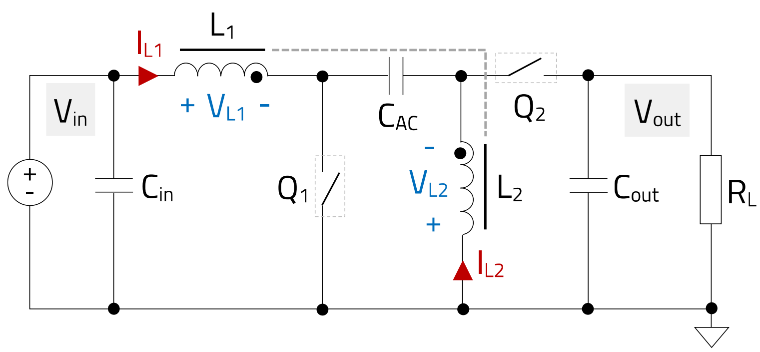


Fig. 1. SEPIC power stage reference schematic.

In steady-state operation, both windings are driven by rectangular-shaped voltage waveforms (Fig. 2).

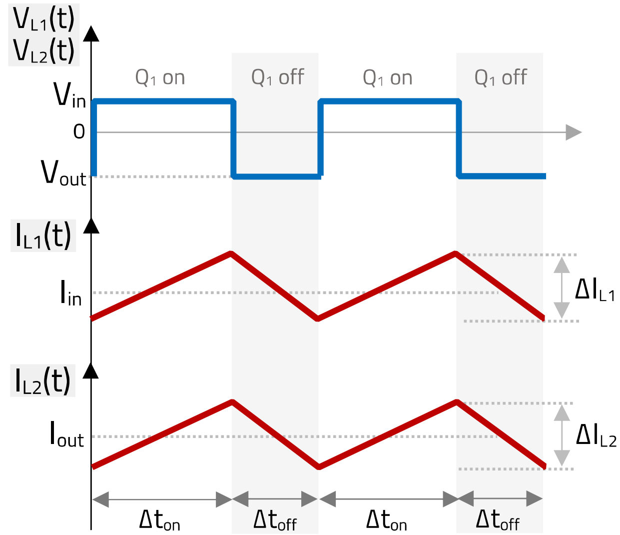


Fig. 2. Inductor waveforms in a CCM SEPIC.

During the conduction time (Δt_{on}) of the control transistor Q_1, the input voltage is applied across each winding and the current rises with (nearly) a first-order slope. Thus, the common expression relating voltage and current in an inductor from equation (1) can be replaced by that of equation (2), where the inductor current ripple is approximated as a first-order function of the constant volt-second applied across its self-inductance L during Δt_{on}:

$$v_L(t) = L \cdot \frac{di_L(t)}{dt} \qquad (1)$$

$$V_{in} \cdot \Delta t_{on} = L \cdot \Delta I_L \qquad (2)$$

From Ampere's law and with the above approximation, the amplitudes of the ripple current and of the generated magnetic flux on a winding are related as in equation (3), where N is the winding's number of turns:

$$\Delta\Phi_{L\_uncoupled} = \frac{L}{N} \cdot \Delta I_{L\_uncoupled} \qquad (3)$$

Based on Faraday's law, the change of magnetic flux ($\Delta\Phi_L$) during Δt_{on} is related to the volt-second applied across the specific inductor winding as follows:

$$\Delta\Phi_{L\_uncoupled} = \frac{V_{in} \cdot \Delta t_{on}}{N} \qquad (4)$$

Fig. 3 shows both inductor coils wound around a toroidal core. For the analysis, both windings are considered to have exactly the same number of turns ($N_1 = N_2 = N$) and physical geometry, while being driven by equal volt-second sources ($VT_1 = VT_2 = VT = V_{in} \cdot \Delta t_{on}$).

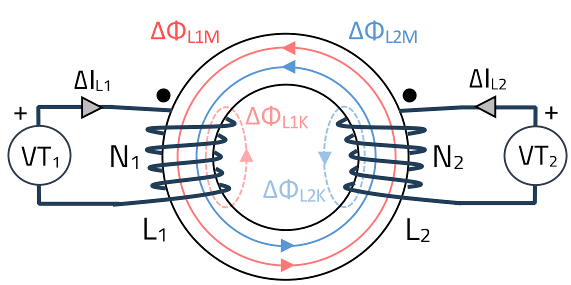


Fig. 3. Coupled inductor windings driven by volt-second sources.

Observe how the winding polarity arrangement causes the magnetic flux generated by one winding to be in phase with that generated by the other winding, both effectively adding up in the core. Considering a very tight magnetic coupling and thus neglecting the leakage flux (i.e. $\Delta\Phi_{L1K} + \Delta\Phi_{L2K} << \Delta\Phi_{L1M} + \Delta\Phi_{L2M}$), the total magnetic flux enclosed by a winding is now composed of that generated by the winding itself plus that generated by the other winding:

$$\Delta\Phi_{L1\_coupled} = \Delta\Phi_{L2\_coupled} \approx \Delta\Phi_{L1M} + \Delta\Phi_{L2M} \qquad (5)$$

However, the same volt-second is still being applied across each winding as when they are uncoupled ($V_{in}\cdot\Delta t_{on}$). Therefore, to satisfy equation (4), the magnetic flux variation in the core for coupled windings needs to be the same as when they are uncoupled:

$$\Delta\Phi_{L1M} + \Delta\Phi_{L2M} = \Delta\Phi_{L\_uncoupled} \qquad (6)$$

As both windings are identical in this analysis:

$$\Delta\Phi_{L1M} = \Delta\Phi_{L2M} = \frac{\Delta\Phi_{L\_uncoupled}}{2} \qquad (7)$$

From equation (3), the condition in (7) is satisfied if the ripple current amplitude on one winding when it is coupled is half of that when it is uncoupled, as below:

$$\Delta\Phi_{L1M} = \frac{L}{N} \cdot \frac{\Delta I_{L1\_uncoupled}}{2} \qquad (8)$$

$$\Delta\Phi_{L2M} = \frac{L}{N} \cdot \frac{\Delta I_{L2\_uncoupled}}{2} \qquad (9)$$

Based on this analysis, it can be understood how tightly coupling the windings while keeping the same number of turns as when they are uncoupled results in half the ripple current amplitude for the same operating conditions. As ripple current and inductance are inversely proportional per equation (2), this also means that a coupled inductor with a very high coupling factor will just require half of the inductance on each winding to obtain the same ripple current amplitude as in the uncoupled case.

So far, this analysis has considered the extreme cases, with either no coupling or with very tight coupling, but what happens in between, when the windings are simply loosely coupled?

Impact Of Coupling Factor On Ripple Current

So far, the winding's magnetic flux which is not coupled to the other winding ($\Delta\Phi_{LxK}$), represented as leakage inductance (L_K), was considered negligible compared to the magnetic flux coupling to the other winding ($\Delta\Phi_{LxM}$), modelled as magnetizing inductance (L_m). Fig. 4 shows the equivalent transformer T-model of the representation of Fig. 3, this time including the influence of the leakage flux, where

L_{1K} = leakage inductance of winding 1 due to $\Delta\Phi_{L1K}$

L_{1m} = magnetizing inductance of winding 1 due to $\Delta\Phi_{L1M}$

L_{2K} = leakage inductance of winding 2 due to $\Delta\Phi_{L2K}$

n = N_2/N_1 = turns ratio

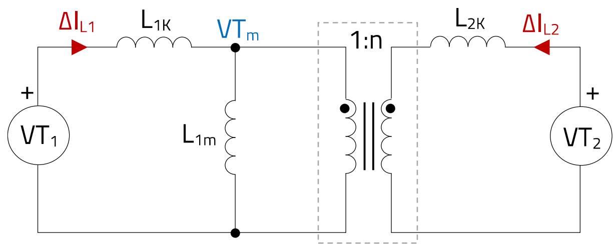


Fig. 4. Equivalent T-model of coupled windings.

Considering a turns ratio of n = 1 and both windings identical, the amount of leakage flux on each winding will be the same (i.e. $L_{1K} = L_{2K} = L_K$), and so will the magnetizing flux (i.e. $L_{2m} = L_{1m} = L_m$). With this, the equivalent circuit schematic of Fig. 4 can be simplified to that of Fig. 5 after transferring the elements from the secondary side over to the primary side.

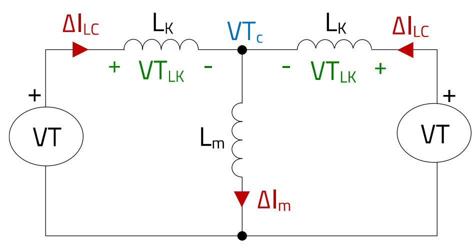


Fig. 5. Equivalent circuit with identical windings.

Observe how, as long as both sides of L_m are identical, the contribution from each side to the volt-second applied across the magnetizing inductance (VT_C) will be the same, as below:

$$VT_{C1} = VT_{C2} = \frac{L_m//L_K}{L_K+L_m//L_K}\cdot VT \qquad (10)$$

where

$$VT_C = VT_{C1}+VT_{C2} \qquad (11)$$

Simplifying equation (10), the following expression is reached:

$$VT_{C1} = VT_{C2} = \frac{1}{\left(2+\frac{L_K}{L_m}\right)}\cdot VT \qquad (12)$$

To simplify equation (12) further, the definition of coupling factor must be revisited. The coupling coefficient of a winding is the ratio obtained by dividing the portion of magnetic flux generated by that winding which is fully contained within the core by the total magnetic flux generated by the winding. Based on this, the coupling coefficient of each winding in our analysis can be expressed as

$$k_1 = \frac{L_{1m}}{L_{1m}+L_{1K}} = \frac{1}{1+\frac{L_{1K}}{L_{1m}}} \qquad (13)$$

$$k_2 = \frac{L_{2m}}{L_{2m}+L_{2K}} = \frac{1}{1+\frac{L_{2K}}{L_{2m}}} \qquad (14)$$

Once the two windings are magnetically coupled, the resulting coupling factor between them can be obtained:

$$k = \sqrt{k_1\cdot k_2} \qquad (15)$$

The expression in equation (15) shows how the coupling factor alone does not provide specific information regarding the construction of the windings, especially regarding where the leakage flux is located: if mostly in one winding or distributed in both windings with equal or different amounts. However, in the case of our analysis, both windings are considered identical, and then

$$k_1 = k_2 = k = \frac{1}{1 + \frac{L_K}{L_m}} \tag{16}$$

From (16), the volt-second contributed by each winding across the magnetizing inductance in the previous equation (12) can now be expressed as a function of the coupling factor, as follows:

$$VT_{C1} = VT_{C2} = \frac{1}{\left(1+\frac{1}{k}\right)} \cdot VT \tag{17}$$

The ripple current amplitude on each winding is also the same in this case, and given as

$$\Delta I_{L1} = \Delta I_{L2} = \Delta I_{LC} = \frac{VT}{L_m} \cdot \frac{1}{\left(1+\frac{1}{k}\right)} \tag{18}$$

By following a similar analysis, the ripple current through one of the windings when the other one is disconnected (uncoupled) is calculated as

$$\Delta I_{1U} = \frac{VT}{L_K + L_m} = \frac{VT}{L_m} \cdot k \tag{19}$$

Finally, from equations (18) and (19), the ripple current ratio between coupled and uncoupled windings is obtained:

$$\frac{\Delta I_{1C}}{\Delta I_{1U}} = \frac{1}{1 + k} \tag{20}$$

This non-linear relationship is plotted in Fig. 6. Observe how, for a perfect coupling factor of k = 1, the ripple current effectively reduces to half. But as the coupling factor decreases (i.e. the leakage inductance increases), so does the attainable ripple current reduction, with the ripple current approaching the same level as in the uncoupled case as k nears zero.

However, note how, even for a rather low coupling factor of k = 0.5 (i.e. with equal leakage and magnetizing flux on a winding), the ripple current amplitude when coupled is still lower than 70% of that of the uncoupled case.

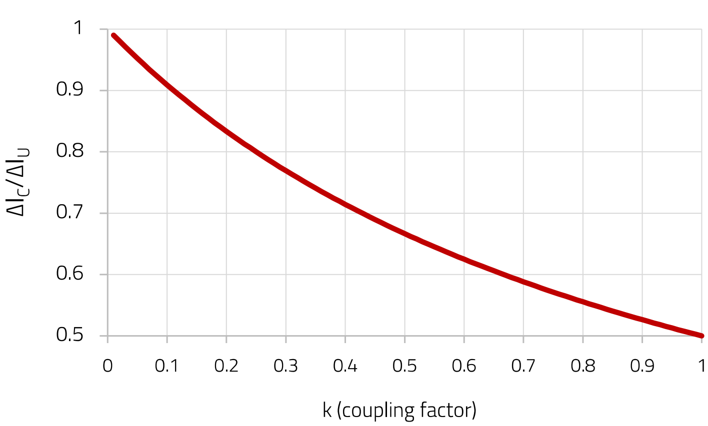


Fig. 6. Current ripple ratio versus coupling factor.

Analogous to the expression in equation (20), the ratio of the winding inductances required to obtain the same ripple current amplitude in the coupled and uncoupled cases can be equally expressed as a function of the coupling factor, as

$$\frac{L_{1C}}{L_{1U}} = \frac{1}{1 + k} \tag{21}$$

Equation (21) shows again how, with a coupling factor of 1, the required inductance can be reduced by half.

As an example of the above, consider a SEPIC performing a 18-V to 12-V conversion, with a load current of 4 A and a switching frequency of 500 kHz. Winding inductances are 10 µH and other component values are as shown in the LTspice simulation schematic on the left in Fig. 7, whereas the winding's currents for coupling factors of k = 0, k = 0.4 and k = 0.9 are shown on the right in Fig. 7.

For the uncoupled case with k = 0, the ripple current amplitude is ΔI_{1U} = 1.5 A. Taking this as a reference and based on equation (20), the calculated ripple current reduction factors when coupling the windings for k = 0.4 and k = 0.9 are 0.72 and 0.53, respectively. These, in turn, correspond to ripple current amplitudes of 1.09 A and 0.79 A, in close agreement with simulated results.

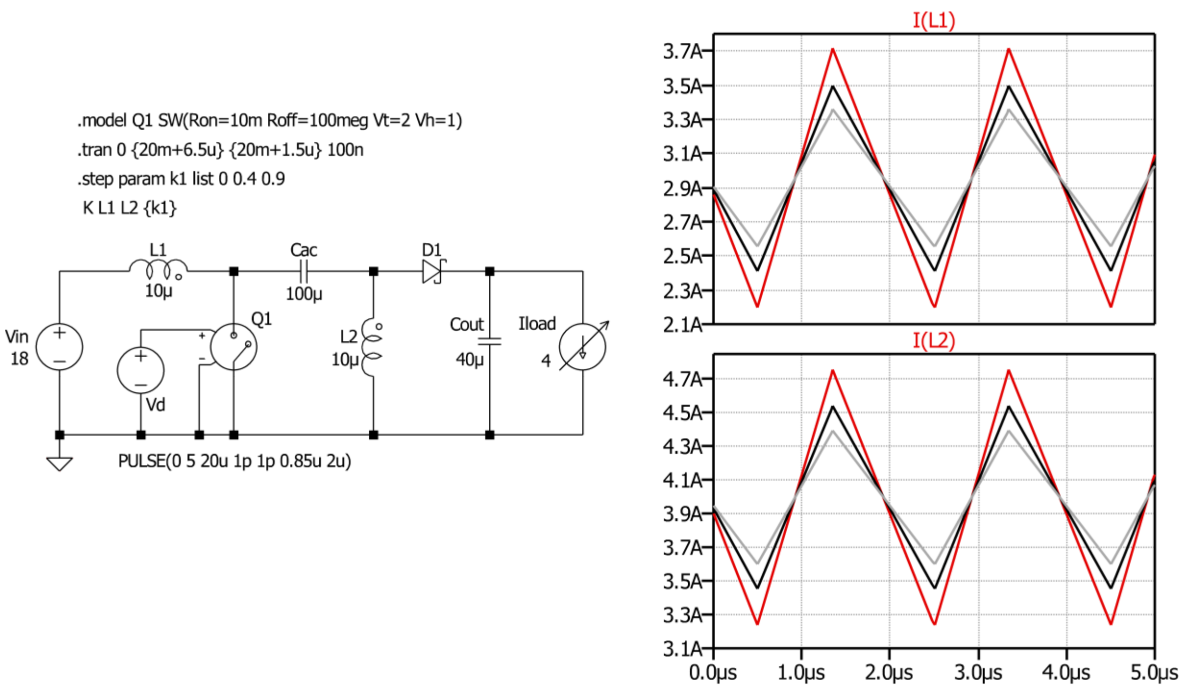


Fig. 7. LTspice simulation of SEPIC example with k = 0 (red), k = 0.4 (black) and k = 0.9 (grey).

Advantage Of Coupling The Windings: A Real Example

By reducing the ripple current amplitude, the ac-related losses are also reduced. This means that a higher efficiency can be achieved when coupling the windings while keeping the same magnetic component size compared to the uncoupled case. Alternatively, this also implies that the component size could be reduced while keeping a similar efficiency.

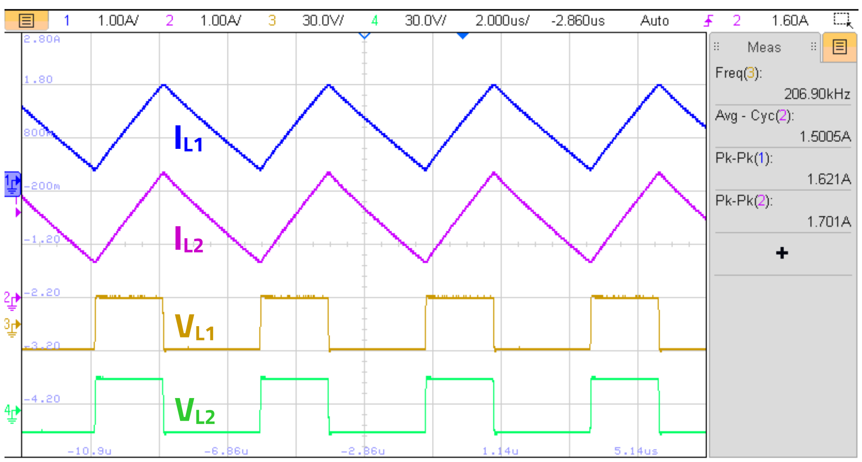
As an example, consider a SEPIC operating at an input voltage of 18 V, an output voltage of 12 V, a load current of 1.5 A and a switching frequency of 200 kHz. The winding inductance is 22 µH and equal for both, the coupled and the uncoupled case. To obtain a representative result, similarly constructed parts must be compared (i.e. same core shape, magnetic material and winding style).

Examples of such equivalent parts are the WE-DD coupled inductor and the WE-PD single inductor (standard version) series from Würth Elektronik, both built with similar ferrite-based shielded drum cores and offered in different size variants. From these, the WE-DD 1280 has a volume of 1328 mm^3, while for the WE-PD 1245 variant this is 648 mm^3, resulting in a total effective volume of 1296 mm^3 for the uncoupled solution. Thus, the total volume of the magnetics is practically the same in both cases, while still keeping a similar winding DCR of around 60 mΩ.

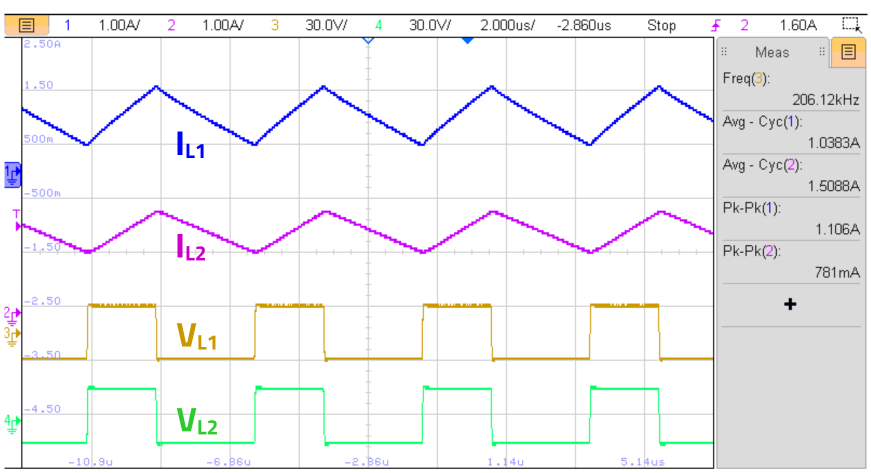
Based on this, the selected parts are the WE-DD 1280 744873220 for the coupled solution and two WE-PD 1245 7447715220 units for the uncoupled one (Fig. 8).

Fig. 8. The two WE-PD 1245 single inductors (left) and the WE-DD 1280 coupled inductors (right) are roughly equivalent in volume, and feature similar inductances and total DCR. These equivalences enable a valid performance comparison in the SEPIC design example.

The measured current and voltage waveforms of each winding for the uncoupled and for the coupled design are shown in Fig. 9, parts (a) and (b), respectively. In Fig. 9a, a ripple amplitude of around 1.7 A on each winding is observed, resulting in a total combined ripple current of almost 3.4 A. In Fig. 9b, the total ripple amplitude is around 1.9 A, not exactly but close to half of the uncoupled case. However, the ripple current on each winding is different: about 0.3 A higher on the input winding (L_1) than on the output winding (L_2).



(a)



(b)

Fig. 9. Voltage and current waveforms of L_1 and L_2 uncoupled (a) and of L_1 and L_2 coupled (b).

In the analysis, perfectly symmetrical windings and exactly the same volt-second applied across them were considered. But in reality, "imbalances" between the windings will cause such "ripple-current-steering" effect, where one winding carries more ac ripple than the other (see the reference). For the same amount of winding imbalance, this effect becomes more exacerbated the higher the coupling factor is.

Finally, the efficiency comparison between the two solutions is shown in Fig. 10. For these measurements, the same SEPIC board was used, only swapping the inductors. As expected, higher efficiency is achieved with the WE-DD coupled inductor thanks to the lower ac losses, with a difference of up to 4% in the lower current range (below 1 A), where such losses dominate.

As the load current increases, so do the dc losses, and as a result, the efficiency gains of the coupled solution gradually reduce. However, up to 2% higher efficiency is still observed at 2.5 A load, a significant improvement compared to the uncoupled design.

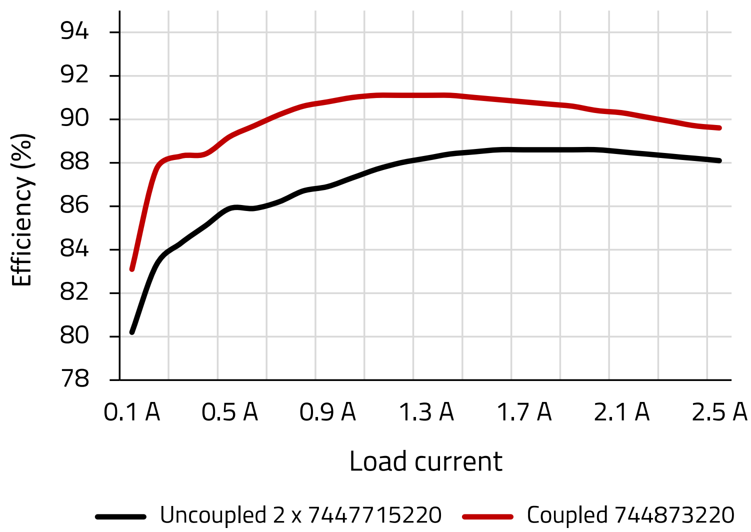


Fig. 10. SEPIC efficiency with coupled and uncoupled windings.

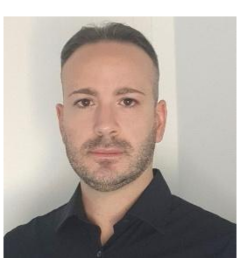
Conclusion

Tightly coupling the windings in a SEPIC can lead to higher efficiency and/or reduced size of the magnetics, but the extent of this advantage reduces as the windings become more loosely coupled. However, this does not mean that a very high coupling factor is always best, as in some particular cases, it could even compromise the correct operation of the converter. Further insights on this and other important design considerations for the SEPIC are provided in Würth Elektronik's new application note ANP135 (see the reference).

Reference

"The SEPIC with coupled and uncoupled inductors" by Eleazar Falco, application note ANP135, Würth Elektronik.

About The Author



Eleazar Falco is currently a senior application engineer at Würth Elektronik. As part of this role, he provides design guidance on the use of different active and passive components in power conversion applications, especially switching power supplies. Some examples are power magnetics, capacitors and optocoupler devices, for which he has authored several publications on topics like gate drivers for SiC devices, dc-dc converter design and feedback loop compensation, among others.

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