ISSUE: November 2025

# Motor Control For Designers (Part 4): PMS Motor Electrical Model

*by Dennis Feucht, Innovatia Laboratories, Cayo, Belize*

Previous articles[1-3] have developed the design equations needed to create an electrical motor model. The resulting model, which will be presented here, is simplified as a magnitude-only model in that it assumes that the torque angle $\delta = \pi/2 = 90°$—that phase control is field-oriented. This is not a major limitation from a design standpoint because magnitude and phase control are usually separate subsystems. (Later, this mini-course will cover phase control.) As noted in Motor Control For Designers (Part 1),[1] the book cited by Krause and Wasynczuk develops a complete model that is taken apart in design for separate magnitude and phase control.

This discussion begins with the introduction of a basic PMS motor model that represents the electrical and mechanical reference frames of the motor. This model, which represents a subset of the Krause and Wasynczuk model, uses a torque-current analogy to convert mechanical parameters to their electrical equivalents. Key terms are defined and the justifications for the choice of this model are explained.

This is followed by an explanation of the torque-speed equation that follows from basic electromagnetics equations as well as related expressions for stall torque and no-load speed. It's further explained how the motor parameters used in these equations are selected when creating a motor design.

In the last section of this article, the concept of Mechanical Ohm's Law is introduced, which allows the PMS motor model to be converted to an all-electrical model that can be used in a circuit simulator to analyze an entire electromechanical system.

## A PMS Motor Magnitude Model

When field-oriented, a PMS motor behaves like a brush motor and is sometimes called a "brushless dc" motor. An electrical circuit model of a PMS motor has electrical and mechanical "sides" which represent the electrical and mechanical reference-frames. (The magnetic field is the third reference-frame that mediates between the sides of the motor.) Fig. 1 shows the basic PMS model.

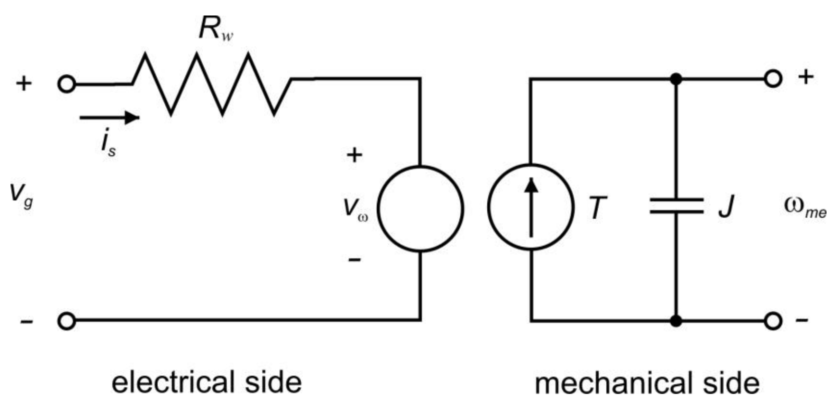


*Fig. 1. Basic PMS motor magnitude-control model with torque-current analogy. Phase is field-oriented. Windings are modeled by series resistance $R_w$ and induced-voltage source $v_\omega$ circuit elements. J is rotor inertia.*

To represent mechanical quantities on the electrical side as elements, the torque-current analogy is used to convert mechanical components to their electrical analogs. Table 1 lists the analogous torque-current components.

We will restrict motor development to rotational motors, though *translational* motors (such as those of levitated trains) move back and forth and deliver force, not torque. This difference has a minor effect on the equations in that conversion parameter $\lambda_{me}$ has different units.

© 2025 How2Power. All rights reserved.

Table 1. Torque-current and force-current analogous elements.

| Electrical | Mechanical |
|---|---|
| Current, *i* | force, *F*<br>torque, *T* |
| Voltage, *v* | speed, *u*, $\omega_{me}$ |
| Circuit flux, $\lambda$ | distance, *r*, $\theta$ |
| Capacitance, *C* | mass, *m*<br>inertia, *J* |
| Inductance, *L* | compliance, *K*, $K_\theta$ |
| Conductance, *G* | damping, *D*, $D_\theta$ |

Electrical-mechanical analogy is possible because linear electrical and mechanical systems have the same form of equations; mechanical and electrical variables correspond in the equations of each and consequently can represent each other. We choose the torque-current correspondence; torque and current are *through* quantities while speed and voltage are *across* quantities.

The intuitive correspondence of the torque-current analogy is consequently more appealing than the alternative torque-voltage analogy. Mechanical and electrical systems in this model are both linear and equivalent for dynamic analysis. Mechanical damping seems like resistance, but it diverts force or torque away from parallel elements as conductance does current in an electrical circuit.

Motors are also generators; the general name for them is *electric machines*. Power conversion is bidirectional. In generator mode, mechanical power applied to the motor shaft converts to voltage $v_\omega$ and current $i_s$ at the electrical port. A motor being decelerated has mechanical energy in the spinning rotor inertia, causing $v_\omega$ to oppose $v_g$. In a *regenerative* motor-drive, this mechanical power is recovered into the $v_g$ source as electrical power.

## Torque-Speed Characteristics

The mechanical performance of motors is expressed in the function $T(\omega_{me})$ and graphed as the torque-speed curve in Fig. 2. Just as oscilloscopes are the basic test instrument for electronic circuits, the central motor instrument is the *dynamometer* which measures torque as a function of motor speed.

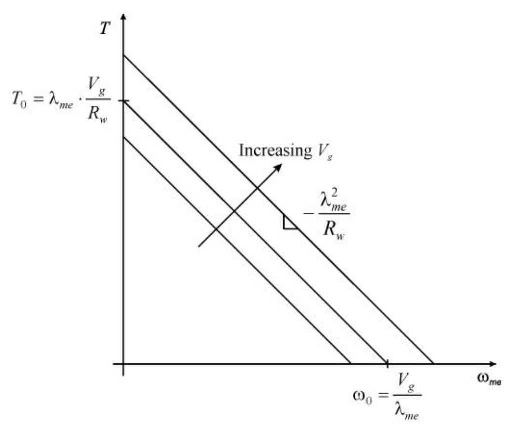


*Fig. 2. Torque vs. speed, $T(\omega_{me})$, plotted with increasing parameter voltage-source amplitude $V_g$. $T(\omega_{me})$ is linear for constant $\lambda_{me}$ (conversion parameter) and $R_w$ (series winding resistance), design parameters which can change the motor torque-speed characteristics.*

© 2025 How2Power. All rights reserved.

We now have enough motor theory to develop performance equations for motors, beginning with $T(\omega_{me})$. For quasistatic (0+ Hz) analysis, the quasistatic electrical model omits electrical inductance and mechanical reactance $J$ while retaining winding resistance $R_w$. From the Fig. 1 motor model, and from KVL, stator current is

$$i_s = \frac{v_g - v_\omega}{R_w} = \frac{v_g - \lambda_{me} \cdot \omega_{me}}{R_w}$$

From the $T(i_s)$ relationship follows $T(\omega_{me})$;

$$T(\omega_{me}) = \lambda_{me} \cdot i_s = \lambda_{me} \cdot \frac{v_g - \lambda_{me} \cdot \omega_{me}}{R_w} = -\frac{{\lambda_{me}}^2}{R_w} \cdot \omega_{me} + \lambda_{me} \cdot \frac{v_g}{R_w}$$

This is the basic *torque-speed equation* of Fig. 2 plotted with stator winding voltage of constant amplitude $V_g$ as parameter. As $V_g$ increases, the plots move away from the origin and mechanical power $P_{me} = T \cdot \omega_{me}$ increases.

The linear plot of the Fig. 2 graph intersects the two axes at points that define the line and are the range extremes of the motor speed and torque; the torque-axis intercept is

$$T_0 = \lambda_{me} \cdot \left( \frac{V_g}{R_w} \right) = \lambda_{me} \cdot I_0$$

which is the *stall torque*.

$T_0$ is maximum torque, at zero speed, where $v_\omega$ = 0 V, and $i_s$ is the maximum of $I_0 = V_g / R_w$. At zero speed, there is no $v_\omega$ to oppose $V_g$ in limiting $i_s$ and maximum current $I_0$ and torque $T_0$ result. If $I_0$ is too large, the magnetic path of the stator saturates and $T(\omega_{me})$ becomes nonlinear. Drive current is typically limited to $I_s < I_0$.

The no-load speed is

$$\omega_0 = \frac{V_g}{\lambda_{me}}$$

$\omega_0$ is maximum mechanical speed at zero torque, where $i_s$ = 0 A, causing no voltage drop across $R_w$, and $v_\omega = V_g$. At $\omega_0$, the induced voltage equals the drive source voltage $V_g$, the voltage across $R_w$ is zero, and no current flows through winding $R_w$; hence no torque can be developed at the no-load speed. (Increasing torque angle to $\delta > 90°$ reduces $v_\omega$, but then the motor magnitude model is no longer valid, lacking field-orientation.)

Motor design basically consists of choosing the motor parameters $\lambda_{me}$ and $R_w$ to give the desired $T(\omega_{me})$. $\lambda_{me}$ depends on motor geometry and $R_w$ on stator turns and wire size. Maximum power $P_{me}$ is at *midscale* ($\omega_0/2$, $T_0/2$);

$$P_{me}(\max) = (\tfrac{1}{2} \cdot T_0) \cdot (\tfrac{1}{2} \cdot \omega_0) = \tfrac{1}{4} \cdot T_0 \cdot \omega_0$$

This corresponds to the electrical circuit load line of a transistor or electron-tube stage where maximum power is dissipated at half the supply voltage and at half the current between cutoff and saturation.

© 2025 How2Power. All rights reserved.

### *Electrical-Mechanical Referral*

The induced-voltage source $v_\omega$ is a portal into the electrical circuit from the mechanical side. The motor model mechanical port has a general mechanical load of $Z_{me}$. An analogous *Mechanical Ohm's Law* is $Z_{me} = \omega_{me}/T$. By the torque-current analogy, speed corresponds to voltage. Similar to winding referral in transformers, $Z_{me}$ is referred to the electrical side through

$$\begin{aligned} v_\omega &= \lambda_{me} \cdot \omega_{me} = \lambda_{me} \cdot (T \cdot Z_{me}) \\ &= \lambda_{me} \cdot (\lambda_{me} \cdot i_s) \cdot Z_{me} = (\lambda_{me}^2 \cdot Z_{me}) \cdot i_s \end{aligned}$$

The mechanical impedance $Z_{me}$ referred from the mechanical to the electrical side is $Z_{me}' = v_\omega/i_s$—an *electrical* impedance; the ′ indicates referral to the other side, making it electrical; hence,

$$Z_{me}' = \lambda_{me}^2 \cdot Z_{me}$$

This side referral is bilateral, as shown in Fig. 3. Without referral, electrical and mechanical ports are as nomenclated on the left. When transformed to the other side, the referrals are indicated as on the right.

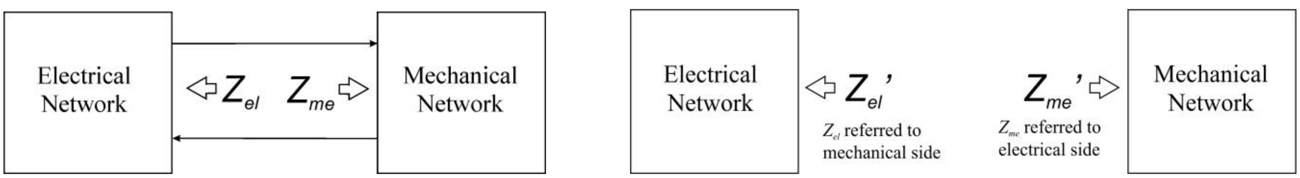


*Fig. 3. Electrical and mechanical ports have impedances as depicted on the left. When either port impedance is referred to the other side of an electric machine, the referral is indicated by a ′ and the impedance is now of the other side as shown on the right.*

Electrical impedance $Z_{me}'$ is $Z_{me}$ as "viewed from" the electrical side through $v_\omega$. $Z_{el}'$ is how the electrical port "looks" from the mechanical side through current source $T$ by $\lambda_{me}^2$ referral of electrical $Z$ into mechanical $Z$.

The equivalent electrical motor model refers the mechanical side of the motor to the electrical side through $v_\omega$ as shown in Fig. 4. And on the mechanical side, mechanical engineers "see" the electrical side through the torque current source $T$.

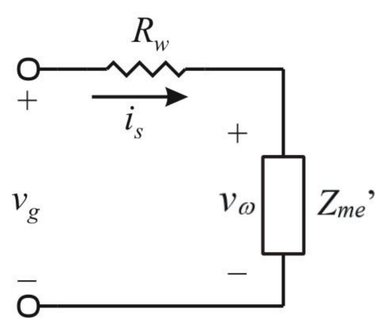


*Fig. 4. $Z_{me}'$ is the equivalent electrical impedance of the entire motor mechanics plus its load $Z_{me}$ as referred by $\lambda_{me}^2$ to an electrical equivalent $Z_{me}'$ across the $v_\omega$ port on the electrical side of the model.*

Table 2 lists some torque-current analogies for basic circuit elements, viewing mechanical elements in equivalent mechanical networks. Mechanical damping is analogous to electrical conductance. Rotational compliance, $K_\theta = \theta/T$, is the inverse of stiffness, and inertia is rotational inertia $J = m \cdot r^2$, where $m$ = mass and $r$ = gyration radius of $m$.

© 2025 How2Power. All rights reserved.

Table 2. $Z_{me} \leftrightarrow \lambda_{me}^2 \leftrightarrow Z_{me}'$.

| $Z_{me}$ | $Z_{me}'$ |
|---|---|
| Inertia, $J$ in N·m·s² units | Capacitance, $C = J/\lambda_{me}^2$ |
| Compliance, $K_\theta$ in 1/N·m | Inductance, $L = \lambda_{me}^2 \cdot K$ |
| 1/Damping, $1/D_\theta$ in 1/N·m·s | Resistance, $R = \lambda_{me}^2 \cdot (1/D_\theta)$ |

Notes: $C$ in F ≡ s/Ω; $L$ in H ≡ Ω·s; $\lambda_{me}^2 \equiv (\text{V}\cdot\text{s})^2$

A useful electrical-mechanical unit conversion is based on equating electrical and mechanical power;

$v_g \cdot i_s = T \cdot \omega_{me} \Rightarrow \text{V}\cdot\text{A} \equiv \text{N}\cdot\text{m}\cdot\text{s}^{-1} \Rightarrow \text{V}\cdot\text{s} = \text{N}\cdot\text{m}/\text{A}$

where $\omega_{me}$ is in $\text{s}^{-1}$ and V·s is the unit of magnetic flux and is equivalent to the (nonobvious) N·m/A. For a unit conversion example, the units of mechanical rotational compliance $K_\theta$ are transformed to the electrical side as

$$K_\theta \text{ units: } \frac{1}{\text{N}\cdot\text{m}} = \frac{1}{\text{V}\cdot\text{A}\cdot\text{s}} \Rightarrow \lambda_{me}^2 \cdot K_\theta \text{ unit: } \frac{(\text{V}\cdot\text{s})^2}{\text{V}\cdot\text{A}\cdot\text{s}} = \frac{\text{V}\cdot\text{s}}{\text{A}} = \Omega\cdot\text{s} = \text{H}$$

The unit of $L$ emerges as the henry, H ≡ Ω·s. Mechanical compliance refers to the electrical side as inductance.

What we now have is an all-electrical motor model that can be input to a circuit simulator and is sufficient for analyzing the entire electromechanical system, including optimization of motor-drive mechanical dynamics with the analogous linear electrically-referred mechanical components. In the next article, we develop from energy instead of the Lorentz force equation the equation for $\lambda_{me}$. Will it be the same?

**References**

1. "Motor Control For Designers (Part 1): Basic Principles Of Motor Theory" by Dennis Feucht, Innovatia Laboratories, How2Power Today, July 2025.

2. "Motor Control For Designers (Part 2): Electromagnetic Force Production In Motors" by Dennis Feucht, Innovatia Laboratories, How2Power Today, August 2025.

3. "Motor Control For Designers (Part 3): Torque-Current Relationship" by Dennis Feucht, Innovatia Laboratories, How2Power Today, October 2025.

**About The Author**

*Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.*

For further reading on motors and motor drives, see "A Practical Primer On Motor Drives".

© 2025 How2Power. All rights reserved.