Winding Turn Lengths And Areas For Magnetic Components (Part 1): Round Windows

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In the design of magnetic components such as transformers and inductors with round or square winding windows, winding lengths and areas are required. Winding resistance varies proportionally with winding length, and for given turns, the wire size that fits the allotted winding area is calculated from the geometrically-derived available area.

In previous articles, I have presented design procedures for determining winding area first, and then winding length. However, determining these parameters can be an iterative process to optimize winding resistance and current density, so the order of calculations can vary after those initial calculations.

Whatever design procedure is chosen, whether it is simple or involves eddy-current geometric constraints of Litz wire bundles with intricate design formulas, winding lengths and areas are in some way a part of the procedure. This article presents generalized formulas for determining winding lengths and areas based on core shapes. The discussion on winding areas includes formulas for achieving equal power density between primary and secondary windings.

The formulas presented in this article and a subsequent one apply to EE, EI, EC, ER, EER, ETD, pot or any other cores with round or rectangular window shapes. This article presents the design formulas and their derivations for round windows. (Note that a previous article presented formulas for toroids.[1])

Winding Turn Length: Round Window

Round or circular turns result from winding windows that are cylindrical, as in pot, barrel, ETD, and EC cores, and rods. The length of wire l_c for a turn of radius r is

$$l_c = 2 \cdot \pi \cdot r$$

A single-strand winding of wire radius r_{cw}, N turns, and winding width w_w has an inner radius of r_i and an outer radius of r_o. The number of turns per layer N_l and number of layers M are

$$N_l = \frac{w_w}{2 \cdot r_{cw}} \;;\; M = \frac{N}{N_l} = \frac{N \cdot (2 \cdot r_{cw})}{w_w}$$

In the most common cases of winding design where the diameter of wires is much less than the winding length —that is, with many turns per layer and $r_{cw} << w_w$—bobbin boundary effects are negligible and wire fitting considerations can be skipped. Then distance between inner layers based on hexagonal layering for the packing configuration is $k_{pl} \cdot r_{cw}$ where $k_{pl} = 1 + \sqrt{3}/2 \approx 1.866$.

(In reference 2, we calculated packing factor, k_{pf}, for a box containing four wires hexagonally layered. We used the dimensions of wires (circles) and box to calculate the packing factor as a ratio of wire area to box area. That yielded the equation:

$$k_{pf} = \frac{4 \cdot \pi \cdot r^2}{(4 \cdot r) \cdot (2 + \sqrt{3}) \cdot r} = \frac{\pi}{2 + \sqrt{3}} \cong 0.842$$

where the numerator represents the wire area and the denominator represents the box area. But since it's not the area that interests us, but rather the height, we divide the denominator $(2 + \sqrt{3})*r$ by the diameter (2r) to obtain the expression for the height parameter, $k_{pl} = 1 + \sqrt{3}/2 \approx 1.866$.)

The wire center of the innermost layer is at $r_i + r_{cw}$; the outermost center is at $r_o - r_{cw}$. The winding height is

$$\Delta r = r_o - r_i = 2 \cdot r_{cw} + (M-1) \cdot [k_{pl} \cdot r_{cw}] = [2 + k_{pl} \cdot (M-1)] \cdot r_{cw}$$

For layer m, the turn radius as measured to the wire center is

$$r(m) = (r_i + r_{cw}) + (m-1) \cdot (k_{pl} \cdot r_{cw}) = r_i + [1 + k_{pl} \cdot (m-1)] \cdot r_{cw} \ , \ m = 1, \ldots, M$$

The average length of a turn of the winding is thus

$$\bar{l}_c = \frac{1}{M} \cdot \sum_{m=1}^{M} 2 \cdot \pi \cdot r(m) = \frac{2 \cdot \pi}{M} \cdot \left(\sum_{m=1}^{M} (r_i + r_{cw} - k_{pl} \cdot r_{cw}) + k_{pl} \cdot r_{cw} \cdot \sum_{m=1}^{M} m \right)$$

Eliminating the summations,

$$\bar{l}_c = \frac{2 \cdot \pi}{M} \cdot \left((r_i + r_{cw} - k_{pl} \cdot r_{cw}) \cdot M + k_{pl} \cdot r_{cw} \cdot \frac{M \cdot (M+1)}{2} \right) \Rightarrow$$

$$\bar{l}_c = 2 \cdot \pi \cdot [r_i + \tfrac{1}{2} \cdot r_{cw} \cdot (2 + k_{pl} \cdot (M-1))]$$

This can be written in the more physically interpretable form of

$$\bar{l}_c = 2 \cdot \pi \cdot \left(r_i + \frac{\Delta r}{2} \right) = 2 \cdot \pi \cdot \left(r_i + \frac{r_o - r_i}{2} \right) = 2 \cdot \pi \cdot \left(\frac{r_o + r_i}{2} \right) = 2 \cdot \pi \cdot \bar{r}$$

For a cylindrical winding window, the average length of a round turn is the length of a turn at the average radius of the winding $\bar{r}$, that of the middle layer.

The average turn length can be expressed in turns by substituting for M,

$$\bar{l}_c = 2 \cdot \pi \cdot \left[r_i + \tfrac{1}{2} \cdot r_{cw} \cdot \left(2 + k_{pl} \cdot \left(\frac{N \cdot (2 \cdot r_{cw})}{w_w} - 1 \right) \right) \right]$$

In addition to the sum of the average turns, each turn has an additional length required for it to move sideways, to span the bobbin width w_w. The spiral travel for M layers adds the length $M \cdot w_w$ for a total winding length of

$$l_w = N \cdot \bar{l}_c + M \cdot w_w = N \cdot (2 \cdot \pi \cdot \bar{r}) + M \cdot w_w \Rightarrow$$

$$\boxed{l_w = N \cdot [2 \cdot \pi \cdot (r_i + \tfrac{1}{2} \cdot r_{cw} \cdot (2 + k_{pl} \cdot (M-1))) + 2 \cdot r_{cw}] \ , \ M = N \cdot (2 \cdot r_{cw}) / w_w}$$

Winding Area: Round Window

For core windows with cylindrical shapes, the window depth is divided into two portions, one for the primary winding(s) and the other for the secondary winding(s). A cross-sectional view is shown in the figure where

$$A_w = A_p + A_s$$

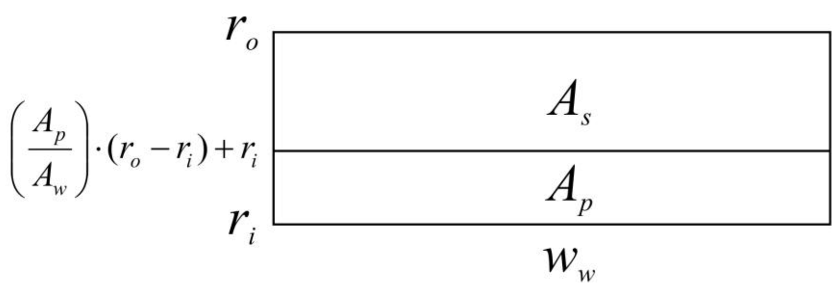

Figure. Area allotment for primary and secondary windings in window of width w_w and height of $r_o - r_i$.

Round-window cross-sectional winding area is

$$A = (r_2 - r_1)\cdot w_w = \Delta r \cdot w_w \;,\; r_2 > r_1$$

and the window winding volume is

$$V = \pi\cdot(r_2^2 - r_1^2)\cdot w_w = 2\cdot\pi\cdot\left(\frac{r_2 + r_1}{2}\right)\cdot(r_2 - r_1)\cdot w_w = 2\cdot\pi\cdot\bar{r}\cdot A$$

The average primary turn length, based on the length derivation and expressed in the fraction of the total winding area that is the primary area A_p/A_w,

$$\bar{l}_{cp} = (2\cdot\pi)\cdot\left(r_i + \frac{(A_p/A_w)\cdot(r_o - r_i)}{2}\right) = 2\cdot\pi\cdot\bar{r}_p$$

For the secondary winding, average turn length is

$$\bar{l}_{cs} = (2\cdot\pi)\cdot\left(\tfrac{1}{2}\cdot\left[r_o + \left(\frac{A_p}{A_w}\right)\cdot(r_o - r_i) + r_i\right]\right) = 2\cdot\pi\cdot\bar{r}_s$$

The ratio of primary and secondary average winding turn lengths is

$$\frac{\bar{l}_{cs}}{\bar{l}_{cp}} = 1 + \frac{1}{\left(\frac{r_i}{\Delta r/2}\right) + \left(\frac{A_p}{A_w}\right)} \;,\; \Delta r = r_o - r_i$$

For constant power-loss density in the winding window,

$$\frac{P_p}{V_p} = \frac{P_s}{V_s} \quad \text{or} \quad \frac{\tilde{i}_p^{\,2}\cdot R_p}{\bar{r}_p\cdot A_p} = \frac{\tilde{i}_s^{\,2}\cdot R_s}{\bar{r}_s\cdot A_s}$$

where R_p and R_s are the primary and secondary winding resistances. Then the ratio of primary to secondary window area is constrained to

$$\frac{A_p}{A_s} = \left(\frac{\tilde{i}_p}{\tilde{i}_s}\right)^2 \cdot \left(\frac{\bar{r}_s}{\bar{r}_p}\right) \cdot \left(\frac{R_p}{R_s}\right) = \left(\frac{\tilde{i}_p}{\tilde{i}_s}\right)^2 \cdot \left(\frac{\bar{r}_s}{\bar{r}_p}\right) \cdot \frac{\rho \cdot \frac{l_p}{A_{cp}}}{\rho \cdot \frac{l_s}{A_{cs}}}$$

where winding resistance $R = \rho \cdot l/A$. Simplifying,

$$\frac{A_p}{A_s} = \left(\frac{\tilde{i}_p}{\tilde{i}_s}\right)^2 \cdot \left(\frac{\bar{r}_s}{\bar{r}_p}\right) \cdot \left(\frac{l_p}{l_s}\right) \cdot \left(\frac{A_{cs}}{A_{cp}}\right)$$

where l_p and l_s are the total winding lengths. Bringing in average turn length, turns ratio n, and packing factor,

$$\frac{A_p}{A_s} = \left(\frac{\tilde{i}_p}{\tilde{i}_s}\right)^2 \cdot \left(\frac{\bar{r}_s}{\bar{r}_p}\right) \cdot \frac{\bar{l}_{cp} \cdot N_p}{\bar{l}_{cs} \cdot N_s} \cdot \frac{\left(\frac{k_{ps} \cdot A_s}{N_s}\right)}{\left(\frac{k_{pp} \cdot A_p}{N_p}\right)} = n \cdot \left(\frac{\tilde{i}_p}{\tilde{i}_s}\right) \cdot \sqrt{\frac{\bar{l}_{cp}}{\bar{l}_{cs}} \cdot \frac{\bar{r}_s}{\bar{r}_p} \cdot \frac{k_{ps}}{k_{pp}}}$$

The average turn length and average radius of turn are related through $l_c = 2 \cdot \pi \cdot r$ so that

$$\frac{\bar{l}_{cp}}{\bar{l}_{cs}} \cdot \frac{\bar{r}_s}{\bar{r}_p} = 1$$

Applying this and the transductor relationship $\tilde{i}_s = n \cdot \tilde{i}_p$, the winding area ratio greatly reduces to

$$\frac{A_p}{A_s} = n \cdot \frac{\tilde{i}_p}{\tilde{i}_s} \cdot \sqrt{\frac{k_{ps}}{k_{pp}}} \approx 1 \ , k_{ps} \approx k_{pp}$$

The fraction of A_w allotted to the primary winding is

$$\frac{A_p}{A_w} = \frac{A_p}{A_p + A_s} = \frac{1}{1 + A_s / A_p} = \frac{1}{1 + \left(\frac{1}{n}\right) \cdot \left(\frac{\tilde{i}_s}{\tilde{i}_p}\right) \cdot \sqrt{\frac{k_{pp}}{k_{ps}}}} = \frac{1}{1 + \sqrt{\frac{k_{pp}}{k_{ps}}}} \approx \frac{1}{2}$$

Then the optimal winding areas are

$$A_p = A_s = A_w / 2$$

Transductors involved in power transfer between windings handle the same power, minus the losses, in primary and secondary windings. Uniform spatial distribution of winding loss in a well-designed transductor has equal primary and secondary winding power losses. Then for equal power-loss density, the volumes, and hence cross-sectional window areas, will be about the same. Magnetic nonlinearity can cause the optimum to shift from equal areas, but it is a broad optimum.

In closing, the design equations presented here assume that wire diameter is a small fraction of winding length. For large wire, area turn fitting considerations might apply. These have been addressed in a previous magnetics article.[3] In part 2 of this article, we'll determine winding lengths and areas for transformers and inductors with square winding windows.

Reference

1. "How To Calculate Toroid Winding Length" by Dennis Feucht, How2Power Today, September 2013.
2. "How To Calculate Winding Packing Factor," by Dennis Feucht, How2Power Today, February 2016. See page 5.
3. "Optimizing Winding Design For Low-Resistance Windings—Selecting Wire Or Bundle Size To Fill The Core Window" by Dennis Feucht, How2Power Today, February 2024.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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