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Motor Control For Designers (Part 5): Deriving Force Production From Magnetic Energy

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In two earlier articles (parts 2 and 3), we used the Lorentz force equation to derive expressions for the force or torque produced by motors. In particular, this led us to a definition of λ_{me}, the conversion factor that relates the electrical and mechanical operation of a motor, and the associated equations using λ_{me} are the basis for a motor model.

The Lorentz force *method* of expressing motor force or torque was a good starting point as it's the easiest to envision geometrically. However, there's an alternative method, which develops motor theory from an *energy* standpoint,

$$\text{Energy Relation: } W = \int \mathbf{F} \bullet d\mathbf{x} = \tfrac{1}{2} \cdot \int_V \mathbf{B} \bullet \mathbf{H} \cdot dv$$

and we explore this method here. Will this yield the same expression for λ_{me}? Let's find out. Perhaps it will also lead to some new revelations about PMS and other motors? In this part, we also return to the subject of motor modeling from part 4 by discussing how to model multiphase windings.

Relating Energy To Magnetic Flux Density And Volume

Power, the rate of change of *energy W*, is

$$P = \frac{dW}{dt} = i \cdot v = i \cdot \frac{d\lambda}{dt} = i \cdot \frac{d(N \cdot B \cdot A)}{dt} = (N \cdot i) \cdot A \cdot \frac{dB}{dt}$$
$$= (H \cdot l) \cdot A \cdot \frac{dB}{dt} = (A \cdot l) \cdot H \cdot \frac{dB}{dt} = V \cdot H \cdot \frac{dB}{dt}$$

From previous construction of design equations, circuit flux λ across the stator windings is the winding turns N times field flux ϕ or $\lambda = N \cdot (B \cdot A)$. Previously, we let the winding area A swept out by the fixed magnet field B_r be variable as dA/dt. If instead we look at a changing magnet flux sweeping across a fixed winding area A, then flux through it varies by dB_r/dt, as in the above equations. The magnetic path begins at the magnets, traverses the air gap, proceeds through the stator "core" or armature to the opposing pole teeth, gap and through the opposite-polarity magnet to close the path. Closed magnetic paths always involve *pairs* of poles.

The winding current referred to the field in the magnetic path is $N \cdot i = H \cdot l$ by Ampere's Law, where l is the magnetic path length and H is the field intensity B/μ. The field has two sources, the magnets and the windings. The property of magnetic material that relates to how much field they can support is *permeability* μ which in steel is high. Thus the field is concentrated (at high B) almost entirely in the teeth and only a little in the windows. (The windows field has leakage inductance in series with R_w as L_w in a more complete dynamic motor model.)

The magnetic path is largely defined by geometric parameters of cross-sectional path area A and path length l, resulting in path volume V. A, l, and hence V are geometric constants. Energy W is the time integral of power;

$$W = \int P(t) \cdot dt = V \cdot \int H \cdot dB$$

Thus energy density,

$$w = \frac{W}{V} = \int H \cdot dB$$

For linear magnetics, μ is constant. Then substituting $B = \mu \cdot H$

$$w = \int \left(\frac{B}{\mu} \right) \cdot dB = \frac{1}{2} \cdot \frac{B^2}{\mu} \Rightarrow W = \frac{B^2 \cdot V}{2 \cdot \mu}$$

The field energy of a motor—the magnetic energy *W* stored in its field—varies with B^2, magnetic-path volume, and path permeability. Fig. 1 plots *B*(*H*) for a magnetic material that is not linear and is *saturating*; *B* is reaching some limiting value from saturation as field intensity *H* increases. Ideally, motor magnetic paths have a linear *B*(*H*) and by design only begin to saturate at their extreme of operation.

The field energy density *w* is shown in Fig. 1 as the area under the curve along the *B*-axis because in the integral for *w*, *H* is the integrand and *B* as incremental *dB* is the variable of integration. Thus, for *H*(*B*) to be integrated along the *B*-axis results in the area under the curve as shown in Fig. 1.

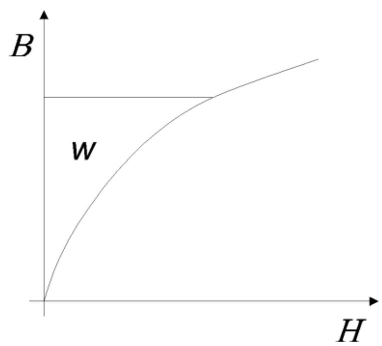

Fig. 1. Graph of B(H) magnetics for a nonlinear material. The curvature shows magnetic saturation (decreasing μ). Whether B(H) is linear or not, the area depicted by w is the field energy density in the material.

Energy Derivation Of λ_{me}

Mechanical energy is

$$W = \int T \cdot d\theta$$

From the previous section, we also know

$$W = \frac{B^2 \cdot V}{2 \cdot \mu}$$

Then, with fixed geometry, volume *V* is constant and torque is

$$T = \frac{dW}{d\theta} = \frac{d}{d\theta} \int_V \frac{1}{2} \cdot B \cdot H \cdot dv = \frac{1}{2} \cdot \frac{d}{d\theta} \int_V \frac{B^2}{\mu} \cdot dv \Rightarrow$$

$$T = \frac{1}{2 \cdot \mu} \cdot \frac{d}{d\theta}(B^2 \cdot V)$$

This equation provides geometric insight for Fig. 2. A magnet N-S edge is sweeping over a one-tooth winding with a magnetic path subtending an arc of θ_t of the tooth. The rotor is moving CCW (right to left) at speed ω_{me}. Stator current i_s in the winding is producing stator magnetic field density B_s with polarity (direction of $\mathbf{B}_s$) as shown. The air gap between magnets and tooth has length *g*, the *width* of the gap.

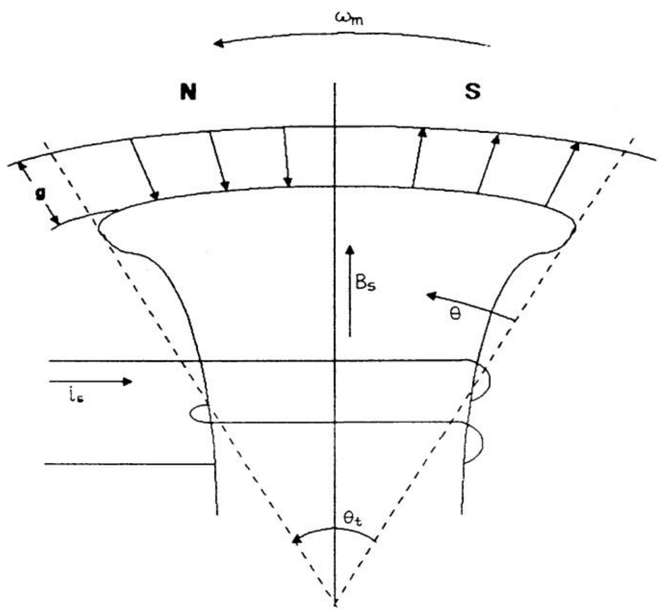

Fig. 2. A N-S magnet edge sweeps over a one-tooth winding subtending arc θ_t. Winding current i_s produces field B_s that adds vectorially to the magnet B_r, and being in alignment with them in the tooth, adds algebraically.

The air gap volume of *B* is

$$V = (r \cdot \theta) \cdot g \cdot l$$

B_s subtracts from magnet flux B_r under N and adds under S. The N-S magnet boundary is at $\theta_t - \theta$ as θ proceeds CCW. Torque produced by energy in the gap is, from the previous torque equation,

$$T = \frac{1}{2 \cdot \mu} \cdot \frac{d}{d\theta}(B^2 \cdot (r \cdot \theta) \cdot g \cdot l)$$

where *r*, *g*, and *l* are all fixed geometric dimensions. Then B^2 underneath the N and S magnets is different, with θ at the N-S edge;

$$T = \frac{r \cdot g \cdot l}{2 \cdot \mu} \cdot \frac{d}{d\theta}(B^2 \cdot \theta) = \frac{r \cdot g \cdot l}{2 \cdot \mu} \cdot \frac{d}{d\theta}[(B_r + B_s)^2 \cdot \theta + (B_r - B_s)^2 \cdot (\theta_t - \theta)]$$

$$= \frac{r \cdot g \cdot l}{2 \cdot \mu} \cdot [4 \cdot B_r \cdot B_s] = 2 \cdot r \cdot g \cdot l \cdot B_r \cdot B_s / \mu$$

θ_t within the derivative is fixed and its derivative is zero, leaving

$$(B_r + B_s)^2 - (B_r - B_s)^2 = 4 \cdot B_r \cdot B_s$$

The gap area subtended by the tooth is A_t. Then applying Magnetic Ohm's Law, stator winding field flux is

$$\phi_s = B_s \cdot A_t = (\mu \cdot H_s) \cdot A_t = \frac{N \cdot i_s}{g} \cdot (\mu \cdot A_t) = \left(\frac{\mu \cdot A_t}{g}\right) \cdot (N \cdot i_s) \Rightarrow$$

$$B_s = N \cdot \left(\frac{\mu}{g}\right) \cdot i_s$$

Substituting for B_s into T, and for p pole-pairs,

$$T = (p \cdot N \cdot 2 \cdot r \cdot l \cdot B_r) \cdot i_s = \lambda_{me} \cdot i_s$$

This is the same result for torque-current conversion as that of the Lorentz-force derivation. Within λ_{me}, $2 \cdot r \cdot l = A$, the magnetic path area, and $B_r \cdot A = \phi_r$, the field flux, and when referred to the electrical side by the total turns for all the pole-pairs of $p \cdot N$ is the circuit flux λ on the electrical side of the motor model. It is denoted as λ_{me} because it refers the mechanical side of the motor to the electrical side as winding-referred circuit flux.

Electromagnetic Energy Conversion

Now that the energy and Lorentz-force derivations have produced the same λ_{me}, the energy method is taken further. The thermodynamic energy polarity convention is $W > 0$ J $\Rightarrow$ W *into* the motor. By conservation of energy, the energy in the magnetic field is the sum of energy into the motor from both electrical and mechanical ports;

$$W_{el} + W_{me} = W_f$$

where W_{el} is electrical energy in the windings, W_{me} is mechanical energy (including that of the reactive mechanical elements J and K) and W_f is the energy of the mediating magnetic field. For linear magnetics, motor magnetic field energy,

$$W_f = \tfrac{1}{2} \cdot L(\theta) \cdot i_s^2$$

where $L(\theta)$ is the motor inductance for a given arc subtended by the magnetic field.

Motor inductance is self-inductance: leakage L + mutual inductance L_{sr} that couples with rotor magnets. Magnets can be modeled as an equivalent constant rotor current i_r through an equivalent rotor winding producing B_r. Then magnetic field energy is

$$W_f(i_s, i_r, \theta_{me}) = \tfrac{1}{2} \cdot L_{ss}(\theta_{me}) \cdot i_s^2 + L_{sr}(\theta_{me}) \cdot i_s \cdot i_r + \tfrac{1}{2} \cdot L_{rr}(\theta_{me}) \cdot i_r^2$$

where θ_{me} is the angle as measured in stationary coordinates attached to the motor stator.

W_{el} in a linear motor—a motor with a linear (constant-μ) magnetic path—goes equally into W_f and W_{me} so that

$$T(\theta_{me}) = \frac{dW_f(i_s, i_r, \theta_{me})}{d\theta_{me}} = \frac{i_s^2}{2} \cdot \frac{dL_{ss}(\theta_{me})}{d\theta_{me}} + i_s \cdot i_r \cdot \frac{dL_{sr}(\theta_{me})}{d\theta_{me}} + \frac{i_r^2}{2} \cdot \frac{dL_{rr}(\theta_{me})}{d\theta_{me}}$$

For a symmetric rotor and stator, L_{ss} and L_{rr} are constant with θ_{me} and thus independent of θ_{me}. That leaves

$$T(\theta_{me}) = i_s \cdot i_r \cdot \frac{dL_{sr}(\theta_{me})}{d\theta_{me}} = \left(\frac{dL_{sr}(\theta_{me})}{d\theta_{me}} \cdot i_r \right) \cdot i_s = \lambda_{me} \cdot i_s$$

(Variable or switched reluctance (VR or SR) motors are based on torque production of varying inductance where one or both of the zero terms above are not zero. Step-motors usually have a small VR torque component of under 5%.) The el ⇔ me conversion parameter λ_{me} relates to the mutual inductance between stator and rotor. Mutual inductance by definition is, in this case, $L_{sr} = \lambda_s / i_r$. Then $\lambda_{me} = T/i_s$ and substituting L_{sr},

$$\lambda_{me} = i_r \cdot \frac{d}{d\theta_{me}} \left(\frac{\lambda_s}{i_r} \right) = \frac{d\lambda_s}{d\theta_{me}} - \frac{\lambda_s}{i_r} \cdot \frac{di_r}{d\theta_{me}} = \frac{d\lambda_s}{d\theta_{me}} - L_{sr} \cdot \frac{di_r}{d\theta_{me}}$$

where the mutual inductance L_{sr} is the stator flux λ_s produced by the equivalent magnet current i_r (that produces the same B_r in an equivalent rotor winding). For the constant i_r of the PM (but not VR) equivalence, then $di_r / d\theta_{me} = 0$, and

$$\lambda_{me} = \frac{d\lambda_s}{d\theta_{me}} = \frac{d\lambda_s}{dt} \cdot \frac{dt}{d\theta_{me}} = v_\omega \cdot \frac{1}{\omega_{me}} = \frac{v_\omega}{\omega_{me}}$$

This is the v_ω el ⇔ me conversion equation and is the same as that derived from the Lorentz-force derivation in part 3, bottom of page 2.

Winding Inductance From λ_{me}

Use of the energy method makes it possible to relate winding inductance and torque. The field variables *B* and *H* relate directly and simply to energy and inductance relates simply to flux, which relates simply to *B* and circuit inductance. While the relationship between inductance and torque is not needed for PMS motor modeling, it will be useful later for understanding variable reluctance motors.

Assume again a PM-equivalent rotor winding and constant-current I_r producing B_r. Then define L_{me} such that

$$\lambda_{me} = L_{me} \cdot I_r = p \cdot N \cdot 2 \cdot r \cdot l \cdot B_r$$

In general, recalling that Magnetic Ohm's Law is $\phi = \mathcal{L} \cdot (Ni)$ where $\mathcal{L}$ = field-referred inductance = $\mu \cdot A / l$,

$$B_r = \frac{\phi_r}{A} = \frac{\left(\frac{\mu \cdot A}{l} \right) \cdot (N \cdot i)}{A} = \frac{\mu}{l} \cdot (N \cdot i)$$

For l = gap width = g and $i = I_r$, then substituting for B_r into λ_{me} for p pole-pairs of N turns,

$$\lambda_{me} = p \cdot N \cdot (2 \cdot r \cdot l) \cdot \left(\frac{\mu}{g} \cdot (p \cdot N \cdot I_r) \right)$$
$$= (p \cdot N)^2 \cdot \frac{\mu \cdot (2 \cdot r \cdot l)}{g} \cdot I_r$$
$$= (p \cdot N)^2 \cdot \mathcal{L}_{me} \cdot I_r$$

where $p{\cdot}N$ is the total number of turns per phase-winding, and $\mathcal{L}_{me}$ is average motor mutual field inductance between stator and rotor. Then the motor mutual circuit inductance is

$$L_{sr} = (p{\cdot}N)^2{\cdot}\boldsymbol{\mathcal{L}}_{me}$$

Series winding inductance that can be added to the motor model is leakage inductance L_{ss}. L_{sr} is in λ_{me} of the induced-voltage v_ω source because it couples between electrical and mechanical sides of the motor, much like the mutual inductance between windings of a transformer. The difference is that in a motor, the equivalent rotor "secondary winding" moves. The total inductance across the PMS winding terminals remains $L_w = L_{ss} + L_{sr}$.

Winding Configurations And Sequencing

Having now developed both Lorentz-force and energy methods for deriving a motor model, we turn our attention to the electrical side and a detail that does not appear in the motor model as such. Some hints have been given previously that the phase-windings of a motor can be *configured* in different ways—that is, they can be interconnected in different configurations.

Phase-windings can each be center-tapped to allow for unipolar-current drive that can create bipolar flux. Each side of the center-tap is an opposite-polarity winding that can be driven by a current controlled by a switch in series with the winding. Depending on which switch is closed, the flux produced is of either polarity.

The circuit can produce bipolar fluxes with unipolar circuit currents. However, only half of the total winding is utilized and torque and energy density of the motor is halved by not driving both half-windings all of the time. Yet this scheme simplifies the drive electronics and can be feasible for smaller motors.

Three-phase motors have three phase-windings, configured to minimize drive switching in two possible three-terminal configurations, shown in Fig. 3. They are driven with bipolar currents and have *dual* configurations: Y (wye, star, T) and Δ (delta, Π).

Y has a central *neutral node* and the phase-windings are connected in series pairs. The Δ configuration has one phase-winding in parallel with two others in series. For either configuration, three terminals can be driven as two (open terminal in DCM, discontinuous conduction mode) or three (all phase-windings conducting in CCM, continuous conduction mode) concurrently. The different drive alternatives result in different induced-voltage waveforms

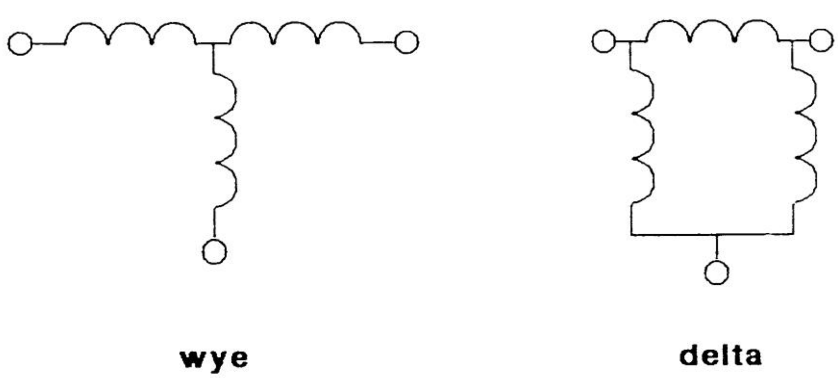

Fig. 3. A three-phase motor has three-phase-windings that can be interconnected or configured as shown. Optionally, the common (neutral) node of the Y (wye) configuration can be brought out of the motor for access.

Phase-windings are each modeled as an induced-voltage generator in series with winding resistance R_w. DCM drive is shown in Fig. 4, where X is the open phase-winding; current in it is discontinuous (0 A) as YZ is driven. The induced voltage across open-circuit YZ terminals (with v_g disconnected) when the rotor is spun is the vector sum of **B** – **C**.

These vectors are the amplitude (magnitude) and phase of periodic waveforms, typically sine-waves, represented as rotating on the vector diagram in Fig. 5. Adding –**B** to **A** results in terminal pair XY voltage of **AB** = **A** – **B**, with magnitude of $\sqrt{3} \cdot A$, lagging **A** by 30° el.

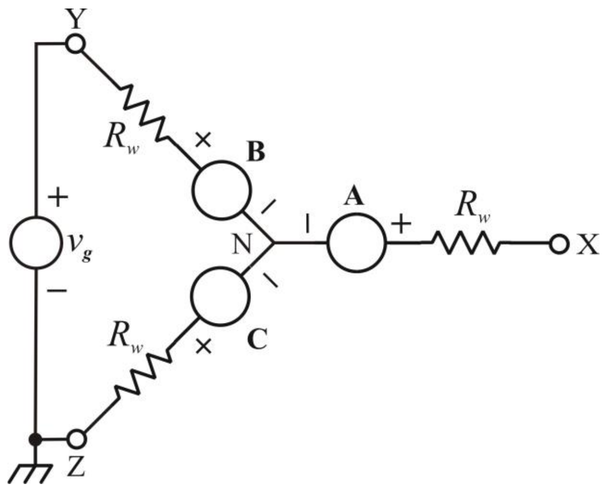

*Fig. 4. Circuit model of a Y-configured motor, with v_ω voltage sources shown as vector phase-winding voltages (to include their phase in the model) of **A**, **B**, and **C**. The terminals are labeled X, Y, and Z. The drive source v_g is shown driving terminal pair YZ ($v_Y - v_Z$) while terminal X is open—a DCM drive of the motor.*

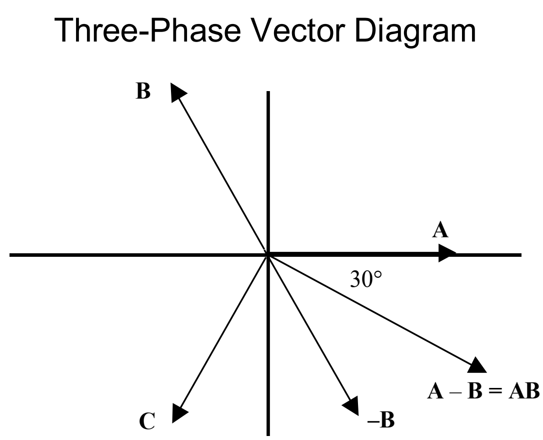

Fig. 5. Vector diagram showing phase explicitly as angles between phase-winding voltages. Vector lengths represent voltage magnitudes; they are equal for an electrically symmetric motor.

In Fig. 4, current from Y to Z (through B and C phase-windings) is

$$i_g = \frac{v_g - (v_B - v_C)}{2 \cdot R_w} = \frac{v_g - v_B + v_C}{2 \cdot R_w}$$

At neutral node N,

$$\begin{cases} v_N = v_g - v_B - i_g \cdot R_w \\ v_N = -v_C + i_g \cdot R_w \end{cases}$$

From the symmetry of the phase-windings,

$$v_A = -(v_B + v_C)$$

Then adding the v_N equations, the voltage at the neutral node is

$$v_N = \tfrac{1}{2}\cdot(v_A + v_g)$$

where v_A is the **A** phase-winding voltage and v_g is the drive voltage applied across terminal pair YZ. The open-terminal voltage of phase-winding **A** is

$$v_X = v_A + v_N = \tfrac{3}{2}\cdot v_A + \tfrac{1}{2}\cdot v_g$$

If node N is available, then $v_A = v_X - v_N$. If not, v_N can be synthesized by connecting equal-value resistors to the three terminals. The resistors form equal × ⅓ voltage dividers and the voltage at the common node of resistors is

$$v_{N'} = \frac{v_X + v_Y + v_Z}{3}$$

Drive to the phase-windings is sequenced by switching (*commutating*) connections to the three pairs of terminals in phase intervals or *steps*. For the drive step shown above, $v_Y = v_g$ and $v_Z = 0$ V. Thus

$$v_{N'} = \tfrac{1}{3}\cdot(v_X + v_g + 0\,\mathrm{V}) = \tfrac{1}{3}\cdot[(\tfrac{3}{2}\cdot v_A + \tfrac{1}{2}\cdot v_g) + v_g] = \tfrac{1}{2}\cdot(v_A + v_g)$$

Substituting for v_X,

$$v_X - v_{N'} = (\tfrac{3}{2}\cdot v_A + \tfrac{1}{2}\cdot v_g) - (\tfrac{1}{2}\cdot(v_A + v_g)) = v_A$$

Voltage across the open phase-winding terminal of X is that of its source v_A and hence $v_{N'} = v_N$. Therefore having N accessible via a fourth wire is not essential because it can be synthesized with three equal-value resistors.

Phase-winding voltages are represented as vectors as indicated in bold on Fig. 4 and shown in Fig. 5. The CCW phase sequence is ACB because the vectors rotate with θ and in sequence cross 0° (where **A** is in Fig. 5) in that order. The vectors are rotating against the fixed stator coordinate frame. For CW rotation, it is ABC; any sequence permutation has opposite direction of rotation. Three terminals, taken in pairs result in six permutations:

AB, AC, BC, BA, CA, CB

The phase diagram of Fig. 6 shows the phase-winding voltages, **A**, **B**, and **C** as vectors at their 0° positions relative to **A**, with positive zero-crossings when rotating CCW.

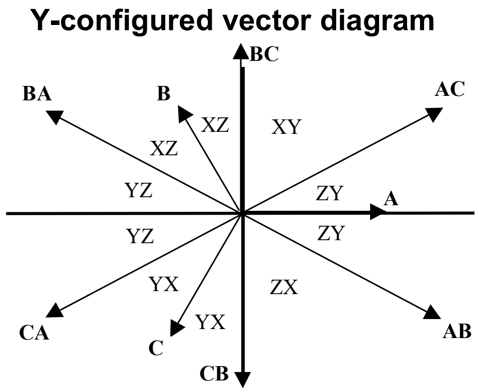

Fig. 6. Complete set of vectors for Y-configured three-phase motor, showing intervals (steps) between vectors of terminal-pair zero-crossing induced voltages that can be detected for phase control (step sequencing).

Drive is applied for a duration of ±30° el around each of the six phase-winding induced-voltage peaks, from 60° to 120° of electrical phase, centered at 90° el. Each of the six steps is driven for an interval of 60° el. In Fig. 6, the XY drive step occurs from 30° to 90° relative to **A**.

The next drive-step, XZ, begins at 90°, coinciding with 0° of **BC**. Thus, a comparator sensing the positive zero-crossing (+zc) of BC can advance the switch sequencing to drive XZ = AC; **AC** had a +zc (with CCW rotation) 90° before the center of the XZ interval where v_{AC} peaks at the +zc of **B**, midway through the XZ drive step. Understanding phase control begins with an understanding of Fig. 6.

Six-step phase control is a simple, low-cost scheme that uses comparators to detect the zero-crossings of the terminal-pair voltages. This scheme will be developed in more detail later. Continuous phase control is better but involves more electronics. Six-step control for many low-cost applications is an optimal design alternative.

In the next part, we'll develop a fuller understanding of three-phase motor winding phase, what voltages occur across pairs of terminals in the Y and Δ configurations, how to relate them, and how to drive the windings.

References

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About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For further reading on motors and motor drives, see "A Practical Primer On Motor Drives".

