Inductor Analytical Model And Its Inductance Dependence On Current

by Gregory Mirsky, Design Engineer, Deer Park, Ill.

Existing models of inductors are mainly intended for usage in simulation programs and do not have all of the inductor's parameters that have to be considered. This article offers an analytical expression for an inductor's inductance. The method described in this article and the supporting plots should significantly facilitate electronic designs incorporating inductors.

The analysis is based on an analytical hysteresis loop equation, which may be familiar to the readers from my prior work (see the reference). In order to make the discussion easy to follow, I've repeated the most important results from my previous relevant publications.

The goal of this publication is to obtain the dependence of the inductor's inductance on the current through the inductor. The resulting formula looks pretty bulky but it accurately reflects the inductance vs. current relationship, which will help designers select the appropriate inductors for their applications.

Hysteresis Loop Expressions As The Starting Point

Analytical expressions obtained earlier in the reference, conveniently describe a magnetic core hysteresis loop using just inductor-plausible data sheet parameters.

$$B_1(H_x) = B_{sat}\left[\frac{2}{1+e^{\frac{-(H_x-H_c)}{H_0}}} - 1\right] \qquad (1)$$

$$B_2(H_x) = B_{sat}\left[\frac{2}{1+e^{\frac{-(H_x+H_c)}{H_0}}} - 1\right] \qquad (2)$$

Here B_{sat} is the core saturation flux density, H_C is the core coercive force, and H_0 is a virtual magnetic field strength, the expression for which will be determined later. H_X is the present magnetic field strength. $B_2(H_X)$ and $B_2(H_X)$ are values for present flux densities for both branches of the hysteresis loop. The B_{sat} and H_C terms are taken from the cores' data sheet parameters.

The median value of the hysteresis loop can be obtained by

$$B(H_x) = \frac{B_1(H_x)+B_2(H_x)}{2} \qquad (3)$$

It is also interesting and useful to get a derivative of the above expression, which represents a change of the core permeability $\mu(H_X)$ along the hysteresis loop. This will allow us to define the value of H_0.

Therefore core permeability is

$$\mu(H_x) = \frac{d}{dH_x}B(H_x)$$

Meanwhile, $\mu(H_X)$ can also be expressed as

$$\mu(H_x) = \mu_0 \cdot \mu_r(H_x)$$

where μ_0 is a free-space absolute permeability (one of the World Constants) and $\mu_r(H_X)$ is the relative permeability of the inductor core material.

But starting with the equation for permeability based on differentiation, we have

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$$\mu(H_x) = \frac{d}{dH_x}\frac{B_1(H_x) + B_2(H_x)}{2}$$

Then, substituting the expressions for $B_1(H_x)$ and $B_2(H_x)$ from (1) and (2) yields

$$\mu(H_x) = \frac{d}{dH_x}\frac{B_{sat}\left[\frac{2}{1 + e^{\frac{-(H_x - H_c)}{H_0}}} - 1\right] + B_{sat}\left[\frac{2}{1 + e^{\frac{-(H_x + H_c)}{H_0}}} - 1\right]}{2}$$

which becomes

$$\mu(H_x) = \frac{B_{sat} \cdot e^{\frac{H_C - H_X}{H_0}}}{H_0 \cdot \left(e^{\frac{H_C - H_X}{H_0}} + 1\right)^2} + \frac{B_{sat} \cdot e^{-\frac{H_C + H_X}{H_0}}}{H_0 \cdot \left(e^{-\frac{H_C + H_X}{H_0}} + 1\right)^2} \qquad (4)$$

Initial permeability μ_{in} is the core permeability at $H_X = 0$ and $H_C = 0$, and can be expressed as:

$$\mu_{ini} = \frac{B_{sat} \cdot e^{\frac{0}{H_0}}}{H_0 \cdot \left(e^{\frac{0}{H_0}} + 1\right)^2} + \frac{B_{sat} \cdot e^{-\frac{0}{H_0}}}{H_0 \cdot \left(e^{-\frac{0}{H_0}} + 1\right)^2}$$

Simplifying, we obtain

$$\mu_{ini} = \frac{B_{sat}}{2 \cdot H_0} \qquad (5)$$

Because this is a data sheet parameter, we can use (5) to solve for H_0:

$$H_0 = \frac{B_{sat}}{2 \cdot \mu_{ini}} \qquad (6)$$

Then plugging (6) into (4) we get

$$\mu(H_x) = \frac{B_{sat} \cdot e^{\frac{H_C - H_X}{\frac{B_{sat}}{2 \cdot \mu_{ini}}}}}{\frac{B_{sat}}{2 \cdot \mu_{ini}} \cdot \left(e^{\frac{H_C - H_X}{\frac{B_{sat}}{2 \cdot \mu_{ini}}}} + 1\right)^2} + \frac{B_{sat} \cdot e^{-\frac{H_C + H_X}{\frac{B_{sat}}{2 \cdot \mu_{ini}}}}}{\frac{B_{sat}}{2 \cdot \mu_{ini}} \cdot \left(e^{-\frac{H_C + H_X}{\frac{B_{sat}}{2 \cdot \mu_{ini}}}} + 1\right)^2}$$

That expression can then be simplified to get

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$$\mu(H_x) = \frac{2\cdot\mu_{ini}\cdot e^{\frac{2\cdot\mu_{ini}\cdot(H_C - H_X)}{B_{sat}}}}{\left[e^{\frac{2\cdot\mu_{ini}\cdot(H_C - H_X)}{B_{sat}}} + 1\right]^2} + \frac{2\cdot\mu_{ini}\cdot e^{-\frac{2\cdot\mu_{ini}\cdot(H_C + H_X)}{B_{sat}}}}{\left[e^{-\frac{2\cdot\mu_{ini}\cdot(H_C + H_X)}{B_{sat}}} + 1\right]^2} \tag{7}$$

From the classical formula for inductance we know,

$$L_m = \frac{S_m \cdot {N_W}^2}{l_m} \mu_0 \cdot \mu_r$$

Assuming (7) as an equivalent for $\mu_0 \bullet \mu_r$, we can get (8),

$$L_m(H_x) = \frac{S_m\cdot {N_W}^2}{l_m} \cdot \left[\frac{2\cdot\mu_{ini}\cdot e^{\frac{2\cdot\mu_{ini}\cdot(H_C - H_X)}{B_{sat}}}}{\left[e^{\frac{2\cdot\mu_{ini}\cdot(H_C - H_X)}{B_{sat}}} + 1\right]^2} + \frac{2\cdot\mu_{ini}\cdot e^{-\frac{2\cdot\mu_{ini}\cdot(H_C + H_X)}{B_{sat}}}}{\left[e^{-\frac{2\cdot\mu_{ini}\cdot(H_C + H_X)}{B_{sat}}} + 1\right]^2}\right] \tag{8}$$

Now recall that the present value of H_X depends on the present magnetizing current, as determined by Ampère's Law, leading to

$$H_X = \frac{I_m\cdot N_W}{l_m} \tag{9}$$

Plugging in (9) into (8), we obtain a formula for inductance as a function of current for any coil having a magnetic core (10):

$$L_m(I_m) = \frac{S_m\cdot {N_W}^2}{l_m} \cdot \left[\frac{2\cdot\mu_{ini}\cdot e^{\frac{2\cdot\mu_{ini}\cdot\left(H_C - \frac{I_m\cdot N_W}{l_m}\right)}{B_{sat}}}}{\left[e^{\frac{2\cdot\mu_{ini}\cdot\left(H_C - \frac{I_m\cdot N_W}{l_m}\right)}{B_{sat}}} + 1\right]^2} + \frac{2\cdot\mu_{ini}\cdot e^{-\frac{2\cdot\mu_{ini}\cdot\left(H_C + \frac{I_m\cdot N_W}{l_m}\right)}{B_{sat}}}}{\left[e^{-\frac{2\cdot\mu_{ini}\cdot\left(H_C + \frac{I_m\cdot N_W}{l_m}\right)}{B_{sat}}} + 1\right]^2}\right] \tag{10}$$

Design Example And Analysis Of Results

To illustrate the use of the inductance expression in (10), let's consider an example in which we determine the winding inductance of a given inductor with the following parameters.

μ_r = magnetic core material relative permeability =1510

S_m = magnetic core cross-sectional area (m^2)= 31 mm^2

μ_{ini} = initial permeability of the magnetic core = $\mu_0 \bullet \mu_r$

l_m = magnetic line length of the magnetic core = 47 mm

H_C = coercive force of the magnetic core = 10 A/m

B_{sat} = permissible magnetic flux density saturation value in the core = 0.35 T

N_W = number of winding turns = 44

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The first five magnetic core parameters are taken from the manufacturer's data sheet.

Using equations (1), (2) and (6), repeated below, we can build the hysteresis loop for this example core, as shown in Fig. 1.

$$B_1(H_x) = B_{sat}\left[\frac{2}{1+e^{\frac{-(H_x-H_c)}{H_0}}} - 1\right] \quad (1)$$

$$B_2(H_x) = B_{sat}\left[\frac{2}{1+e^{\frac{-(H_x+H_c)}{H_0}}} - 1\right] \quad (2)$$

$$H_0 = \frac{B_{sat}}{2\cdot\mu_{ini}} \quad (6)$$

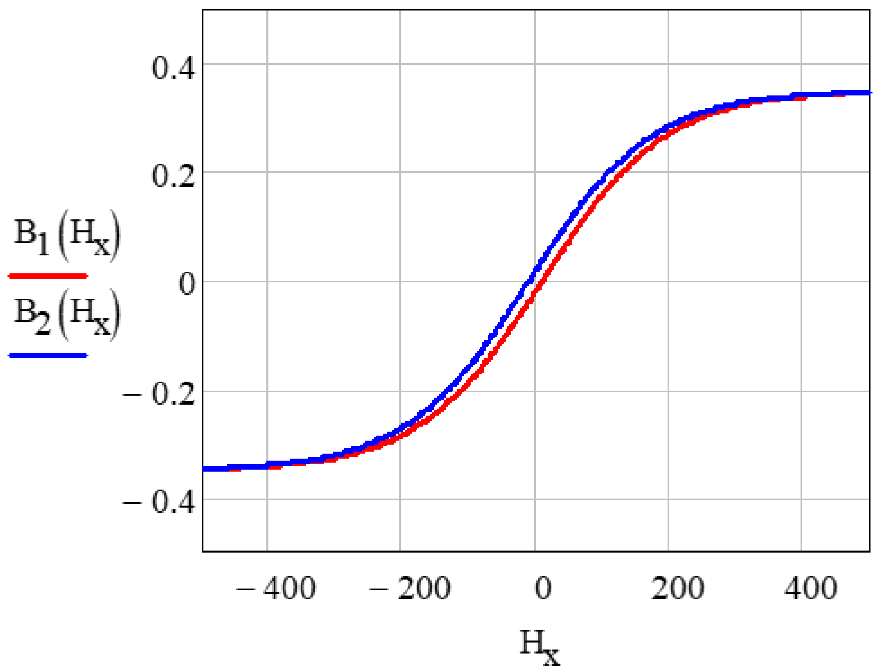

Fig. 1. Hysteresis loop for the example inductor core.

Expressions for the hysteresis loop also allow us to plot a graph for the magnetic core permeability behavior, using (7). The example results are shown in Fig. 2.

$$\mu(H_x) = \frac{2\cdot\mu_{ini}\cdot e^{\frac{2\cdot\mu_{ini}\cdot(H_c-H_x)}{B_{sat}}}}{\left[e^{\frac{2\cdot\mu_{ini}\cdot(H_c-H_x)}{B_{sat}}}+1\right]^2} + \frac{2\cdot\mu_{ini}\cdot e^{-\frac{2\cdot\mu_{ini}\cdot(H_c+H_x)}{B_{sat}}}}{\left[e^{-\frac{2\cdot\mu_{ini}\cdot(H_c+H_x)}{B_{sat}}}+1\right]^2}$$

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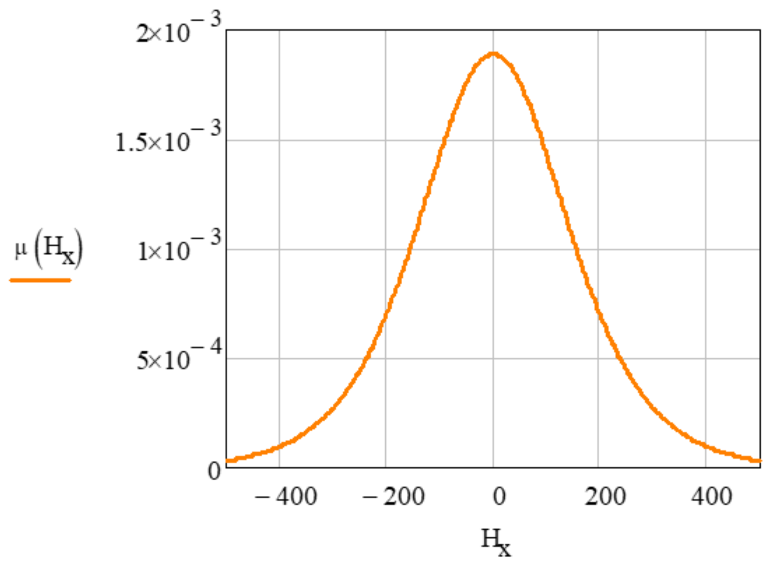

Fig. 2. Core permeability for example inductor.

Finally, using equation (10),

$$L_m(I_m) = \frac{S_m \cdot N_W^2}{l_m} \cdot \left[\frac{2 \cdot \mu_{ini} \cdot e^{\frac{2 \cdot \mu_{ini} \cdot \left(H_c - \frac{I_m \cdot N_W}{l_m}\right)}{B_{sat}}}}{\left[e^{\frac{2 \cdot \mu_{ini} \cdot \left(H_c - \frac{I_m \cdot N_W}{l_m}\right)}{B_{sat}}} + 1 \right]^2} + \frac{2 \cdot \mu_{ini} \cdot e^{-\frac{2 \cdot \mu_{ini} \cdot \left(H_c + \frac{I_m \cdot N_W}{l_m}\right)}{B_{sat}}}}{\left[e^{-\frac{2 \cdot \mu_{ini} \cdot \left(H_c + \frac{I_m \cdot N_W}{l_m}\right)}{B_{sat}}} + 1 \right]^2} \right]$$

we can obtain the inductance for the example inductor as a function of current. Its plot can be found in Fig. 3.

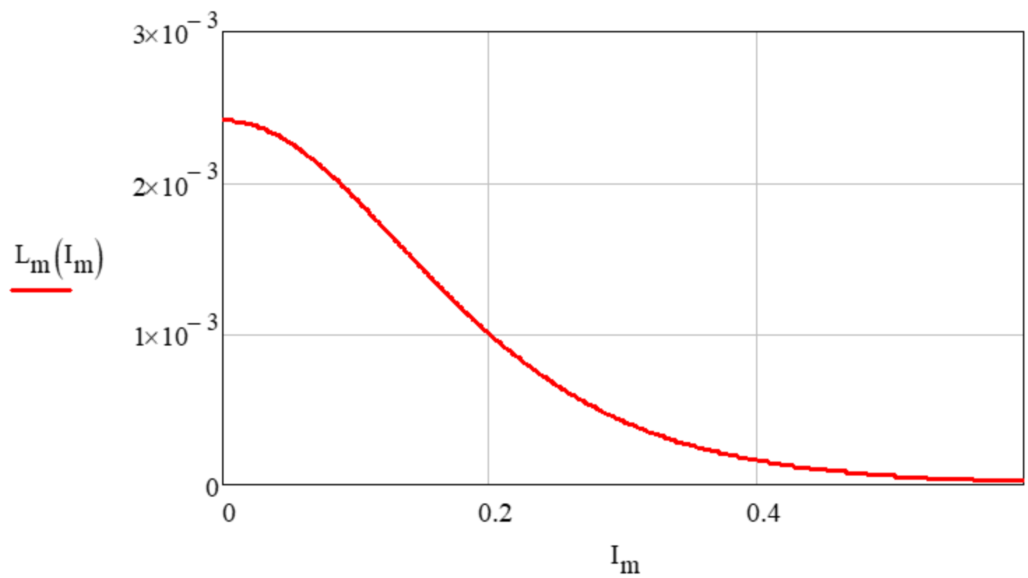

Fig. 3. Inductance of the example inductor as a function of magnetizing current.

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Takeaway

When designing inductors wound on magnetic cores, one should always consider their operating current to keep inductance within admissible limits. At higher operating current and current pulses, inductance may drop down to deep saturation levels and cause electronic device damage.

When selecting inductors for *any* power supplies, even those based on transformers, this recommendation should be considered since these devices utilize inductors as well.

Reference

"The Importance Of Knowing Magnetic Core Saturation Field Strength For Accurate Hysteresis Loss Calculation" by Gregory Mirsky, How2Power Today, January 2025.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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