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Motor Control For Designers (Part 7): Quasistatic And Dynamic PMS Motor Models

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Step-motors are PMS motors with two phase-windings and many pole-pairs. The concepts presented in this part apply to PMS motors of any number of phase-windings by modifying the number of phase-windings, or "*phases*". However, with just two phase windings, they provide a simple starting point for discussion of PMS motor models and a concrete illustration of their use.

A review of the motor electromechanical model from part 4 and torque-speed graphs is followed by some aspects affecting motor-drive design: the voltage drive requirement V_g for full performance.

The dynamic motor model presented in ths part is a more-complete version of the model presented earlier, and allows us to observe the dynamics of the motor in the frequency domain as discussed below in the section on time-constants. We'll be applying this more-complete model going forward in the series. Meanwhile, the quasitatic model, is also useful, allowing us to determine the torque-speed characteristic of the motor.

Basic PMS Motor Model

Step-motor motion performance is specified by ranges of torque and speed. For a given motor, power is also specified as

$$P = T \cdot \omega_{me}$$

where T is torque (in units of N·m) and ω_{me} is the mechanical speed or frequency of the rotor, in s^{-1}. Step-motors are permanent-magnet (PM) synchronous (PMS) motors with many pole-pairs, typically 50. Other "servo" PMS motors usually have one to four pole-pairs. The number of pole-pairs p relates mechanical and electrical frequencies;

$$\omega_{el} = p \cdot \omega_{me}$$

p is also a factor in how much induced voltage is generated by the motor at a given speed. The induced voltage is

$$v_{\omega} = \lambda_{me} \cdot \omega_{me}$$

λ_{me} is the *electromechanical flux* of the motor, the motor field-flux amplitude referred to the mechanical reference frame of the motor. Not only is it the proportionality factor relating induced voltage and speed, the very same parameter also relates torque and stator winding current;

$$T = \lambda_{me} \cdot i_s$$

i_s is the total stator-current magnitude, calculated from the vector sum of the phase-winding currents. Typically, step-motor torque is specified at a given current, such as 2.8 A. This current is assumed to be applied to all phase-windings. For two-phase motors, the currents in the two phases are separated by 90 degrees electrical and the total resultant current vector has a magnitude of $\sqrt{2} \cdot (2.8\ \text{A}) \approx 3.96$ A. A motor specified to have a torque of 2.08 N·m at 2.8 A per phase-winding has an electromechanical flux of

$$\lambda_{me} = \frac{T}{i_s} = \frac{2.08\ \text{N} \cdot \text{m}}{\sqrt{2} \cdot (2.8\ \text{A})} = 0.525\ \frac{\text{N} \cdot \text{m}}{\text{A}} = 0.525\ \text{V} \cdot \text{s}$$

T and i_s are assumed to be peak or magnitude values in λ_{me}, but they can also be calculated in RMS or average values for both T and i_s if it is understood that all T and i_s calculations involving λ_{me} are in those values. Throughout this series, peak or sine amplitude or magnitude values are assumed.

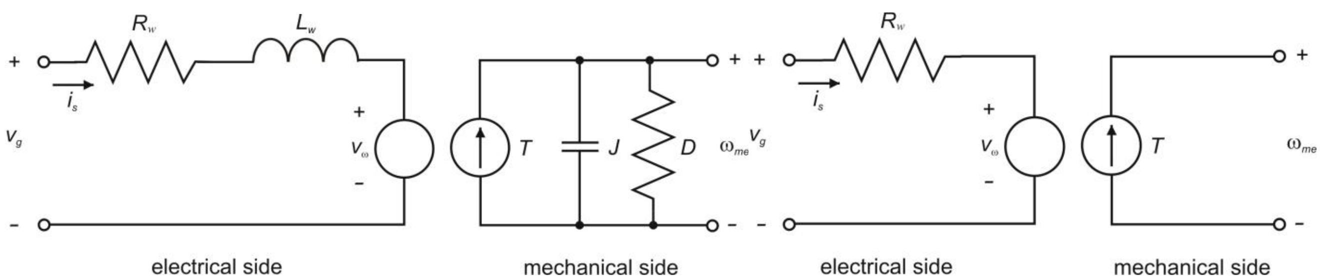
Worth remembering is the unit conversion of N·m/A ≡ V·s which can be recollected by equating units of power:

$$F \cdot u = T \cdot \omega = v \cdot i \;\Rightarrow\; \text{N}\cdot(\text{m/s}) = (\text{N}\cdot\text{m})/\text{s} = \text{V}\cdot\text{A} = \text{W}$$

where u is linear speed, translational power is $P = F \cdot u$ and $P = T \cdot \omega$ is rotational power.

The above equations, combined with the motor parameters of winding resistance R_w and motor drive voltage V_g—the winding terminal voltage—are modeled by the circuit of Fig. 1 by reducing the more complete dynamic model on the left to the quasistatic model on the right. The reduced model has constant voltages and currents, torques and speeds (corresponding to amplitudes of periodic waveforms) when these quantities are referred to the synchronously-rotating reference-frame of motion which is the rotor in a PMS motor.

As we take a position on the rotor and turn with it, these quantities appear constant. They are the magnitudes of the rotating vectors in a vector diagram having vectors rotating with the rotor in its coordinate system. In steady-state and at constant speed, such vectors are called *phasors*. (Most electric machines books present only steady-state theory in phasors.)



(a) dynamic model *(b) quasistatic model*

Fig. 1. Motor electrical model with mechanical-electrical torque-current analogy for converting mechanical elements to corresponding electrical elements. Additional reactive elements are included in the dynamic model (a). (This is a more-complete dynamic model than that presented in part 4 as it adds winding inductance L_w and mechanical damping D.) The torque-speed graphs are based on the quasistatic model, omitting reactances (b).

The quasistatic model applies to a single point of $T(\omega_{me})$ with constant T and ω_{me}. A more inclusive model has reactance, beginning with the mechanical reactance of the rotor inertia J as in the Fig. 1 dynamic model. This motor parameter is a mechanical reactance, the shaft rotational inertia

$$J = m \cdot r_g^2$$

where m is the shaft mass and r_g is the radius of gyration, the radius of the center-of-mass of m. If all the mass were placed at r_g as a thin hoop, then J would result. For non-hoops, J is calculated as an integral over the radius of the rotating object. J is related to mechanical motion as

$$T = J \cdot \alpha_{me}$$

where α_{me} is the rotational mechanical acceleration. Typical units for J are $\text{N}\cdot\text{m}\cdot\text{s}^2 \equiv \text{kg}\cdot\text{m}^2$.

This motor model is decidedly electrical, to facilitate electronic motor-drive design or circuit simulation of a system that is both electrical and mechanical. The mechanical side of the motor is modeled as an electrical circuit using the torque-current (and speed-voltage) analogy. In this analogy, a torque source is modeled as a current source, a "through" quantity, while speed is an "across" quantity as is voltage. A torque-voltage analogy could equally be applied but it loses its intuitive appeal in that through and across quantities are not made analogous.

With these correspondences, it is possible to define mechanical impedance analogously as

$$Z_{me} = \frac{\omega_{me}}{T}$$

Then from the dynamic relationship for *J*,

$$J = \frac{T}{\alpha_{me}} = \frac{T}{\frac{d\omega_{me}}{dt}}$$

In the *s*-domain, this can be expressed as

$$J = \frac{T}{s \cdot \omega_{me}}$$

or analogously,

$$Z_{me} = \frac{\omega_{me}}{T} = \frac{1}{s \cdot J}$$

For the torque-current analogy, then *J* is analogous to capacitance. Furthermore, inductance is analogous to compliance ($L \leftrightarrow K$) and resistance is analogous to 1/damping (1/*D*) or 1/torsional damping for rotational mechanics.

With these analogies, the above circuit model can be solved for $T(\omega_{me})$. The definition of *stall torque* is

$$T(\omega_{me} = 0) = T_0 = \lambda_{me} \cdot I_0$$

where *stall current* is

$$I_0 = V_g / R_w$$

and v_g is held constant as circuit parameter V_g. This is typically the case for motor-drives with H-bridges supplied by a voltage source of V_g. The *no-load* or *base speed* is

$$\omega_{me}(T = 0) = \omega_0 = V_g / \lambda_{me}$$

Then substituting into the $T(\omega_{me})$ equation,

$$T(\omega_{me}) = -\frac{\lambda_{me}^2}{R_w} \cdot \omega_{me} + \lambda_{me} \cdot \frac{V_g}{R_w} = \lambda_{me} \cdot I_0 \cdot \left(1 - \frac{\lambda_{me}^2 \cdot \omega_{me}}{\lambda_{me} \cdot I_0 \cdot R_w}\right)$$

Note that in the above expression for $T(\omega_{me})$, no reactive elements are present. These were eliminated in solving for stall torque and stall current. So now the quasistatic model applies.

$T(\omega_{me})$ reduces compactly to

$$T(\omega_{me}) = T_0 \cdot \left(1 - \frac{\omega_{me}}{\omega_0}\right)$$

The torque-speed function $T(\omega_{me})$ is plotted in Fig. 2 with V_g as a parameter. As V_g increases, the $T(\omega_{me})$ line moves away from the origin while retaining the same slope. Both T_0 and ω_0 increase with increasing V_g.

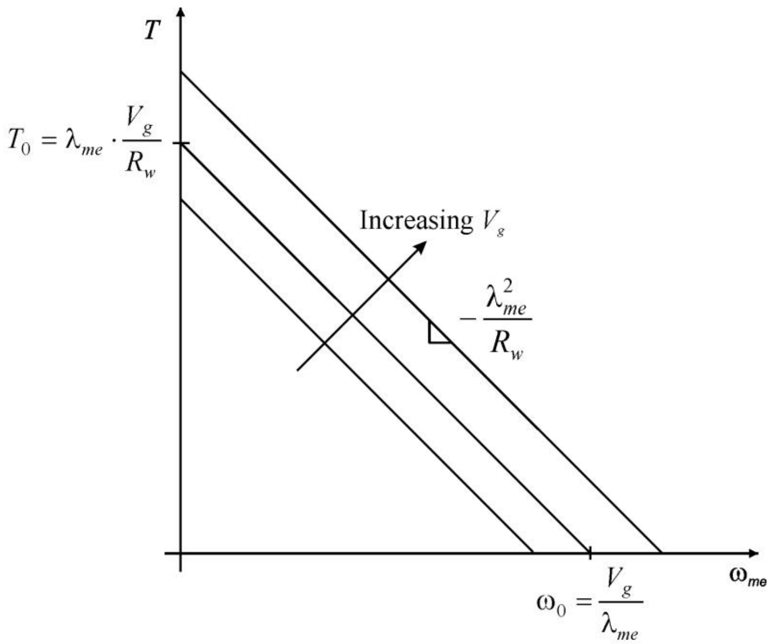


Fig. 2. Graph plotting quasistatic torque $T(\omega_{me})$ with V_g as a parameter. The family of plots are linear and in the first quadrant for motor mode, of positive torque output at the mechanical port and CCW motion. Readers may recall this graph from part 4. The origins of this graph have been explained more fully in this part.

As covered in part 4 under "Electrical-Mechanical Referral," each side of the motor can also be referred to the other through impedance transforms. Z_{me} as defined above is based on the torque-current analogy. Z_{me} can be referred to the electrical side of the motor as Z_{me}' and inversely, electrical impedance Z_{el} can be referred to the mechanical side of the motor as mechanical impedance Z_{el}'.

Z_{me}' is the electrical impedance across v_ω and Z_{el}' is the mechanical impedance loading the torque source. Z_{el} is the electrical impedance across v_ω and Z_{me} is the mechanical load across the T current source.

The relationship between Z_{me} and Z_{me}' can be derived by expressing Z_{me}' across the induced-voltage source as

$$Z_{me}' = \frac{v_\omega}{i_s} = \frac{\lambda_{me} \cdot \omega_{me}}{T / \lambda_{me}} = \lambda_{me}^2 \cdot \frac{\omega_{me}}{T} = \lambda_{me}^2 \cdot Z_{me}$$

This is a mechanical (Z_{me}) to electrical (Z_{me}') impedance transformation analogous to a transformer with turns ratio λ_{me}.

Impedance transformation extends to the dynamic model. A motor winding driven by a source with impedance Z_g is in series with winding series impedance

$$Z_s = R_w + s \cdot L_w$$

Z_s appears on the mechanical side in

$$Z_{el}' = \frac{Z_g + Z_s}{\lambda_{me}^2} = \frac{Z_{el}}{\lambda_{me}^2}$$

For a voltage-source drive of the motor, $Z_g = 0\ \Omega$, and for the quasistatic model,

$$Z_{el}' = \frac{R_w}{\lambda_{me}^2}$$

If R_w is small, voltage-source V_g when referred to the mechanical side of the motor requires much torque to change the speed. This should not be confused as being *stiff* (low compliance); it is not mechanically reactive but is resistive.

In effect, a voltage source shorts the motor windings. Shorted terminals make it hard to turn the shaft because of the counter-torque produced by current v_ω / Z_{el}'. A current-source drive instead allows a speed change with a small torque change; motor open terminals make it easy to turn the shaft.

From the circuit of the quasistatic motor model in Fig. 1, the referred mechanical impedance Z_{el}' is −1/slope of $T(\omega_{me})$, which can be rewritten as

$$T(\omega_{me}) = -\frac{\lambda_{me}^2}{R_w} \cdot \omega_{me} + \lambda_{me} \cdot \frac{V_g}{R_w} = -\frac{1}{Z_{el}'} \cdot \omega_{me} + T_0$$

As the electrical source impedance increases, the slope magnitude of $T(\omega_{me})$ decreases. For a current-source drive, $Z_{el} \to \infty\ \Omega$, and $T(\omega_{me})$ becomes a family of horizontal, constant-torque lines that vary in value with current. By driving the windings with a current source, it is possible to sense across the winding terminals a voltage from which induced voltage v_ω can be recovered and the phase of the motor determined for motor control.

Three Time-Constants

By adding both electrical and mechanical reactances for a more-complete motor model of Fig. 1a (which still assumes field-oriented phase control and is not yet fully complete), we can begin to see the dynamics of the motor from frequency-domain circuit analysis. Motor inertia J is referred to the electrical side of the model as a capacitance of value J / λ_{me}^2. It can be substituted for the induced-voltage source in the model so that circuit analysis includes the mechanics. The damping term D can be treated similarly (Fig. 3).

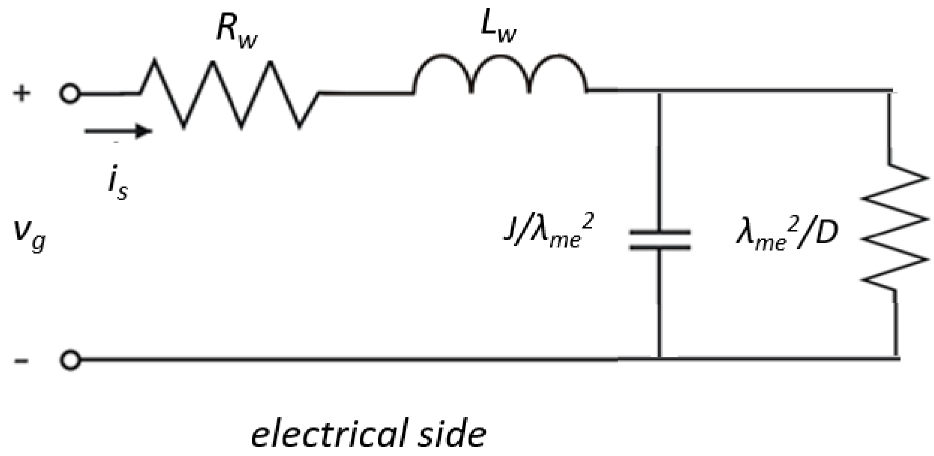


Fig. 3. Dynamic motor electrical model with mechanical parameters referred to the electrical side.

This "mechanical capacitance" resonates with winding series inductance to produce the electromechanical motor resonance at resonant frequency $\omega_n = \omega_{em}$. The impedance plot for the dynamic model of Fig. 1 is shown in Fig. 4. The *electrical* frequency ω_{el} is on the horizontal axis because analysis is from the electrical side of the motor.

On the electrical side of the motor, the series RL of the winding forms an *electrical time-constant* of $\tau_{el} = L_w/R_w = 1/\omega_{el}$. On the mechanical side, a wholly mechanical time-constant analogous to an RC time-constant is $\tau_{me} = J/D = 1/\omega_{me}$ (keeping in mind that D is analogous to conductance).

Fig. 4 depicts time-constants $\omega_{el} < \omega_{me} < \omega_n$ and that 1/damping refers electrically to a much higher resistance than R_w, which is usually the case. In the case that L_w is small, the break frequency at $\omega_{el} = 1/\tau_{el} > \omega_n$ and the resonance disappears as R_w forms a break frequency with J/λ_{me}^2, resulting in an RC time-constant. The *electromechanical time-constant* of $\tau_{em} = \sqrt{[L_w \cdot (J/\lambda_{me}^2)]}$ is a time-constant involving reactances on both sides of the motor. For motion-control design, it is desirable to minimize L_w and eliminate the resonance.

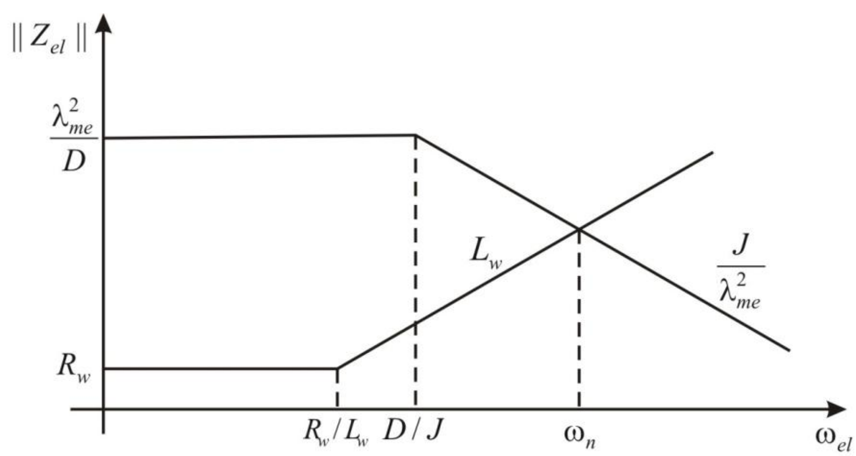


Fig. 4. Impedance magnitude plots of motor model dynamics with series winding inductance L_w and damping D.

The zero and poles of the Fig. 4 graph are included in feedback control of motion and will be considered in the future, in the section of this series on motion control.

Although both quasistatic and dynamic considerations were applied in construction of the motor model, keep in mind that this model does not include phase and is only a magnitude model. Phase is fixed at a torque angle of δ at right angles, where maximum torque is produced.

Even so, this model is sufficient as a foundation for designing the magnitude controller of the motor-drive because, under most circumstances, the phase controller keeps the phase field-oriented where the motor model that has been developed can be applied in design.

References

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About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For further reading on motors and motor drives, see "A Practical Primer On Motor Drives".