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# Motor Control For Designers (Part 8): Step-Motor $V_g$ Range And Efficiency

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In part 7 of this series,[1-7] a more complete version of the dynamic PMS motor model and a quasistatic model of the motor were presented, explaining the origins of the torque-speed graph given earlier. The latter allowed us to begin discussion of the voltage drive requirement $V_g$.

We continue that discussion in this part by determining the voltage drive requirement $V_g$ for the full torque range and the conditions for maximum efficiency $\eta_{max}$. Once again a PMS motor will be used to illustrate the concepts. The default examples are step-motors.

## Vg Range Requirement

One of the design considerations for $v_g$ is the voltage range required of $V_g$ to maintain the desired current. H-bridges are driven by sources in series with winding impedance $Z_s$ and can be made to behave as current sources by feedback control. If a given torque, corresponding to a given current, is commanded at a given speed, and this speed-torque point lies above the $T(\omega_{me})$ line for a given $V_g$, then $V_g$ is insufficient to produce the current corresponding to the commanded torque, and $V_g$ must be increased (Fig. 1).

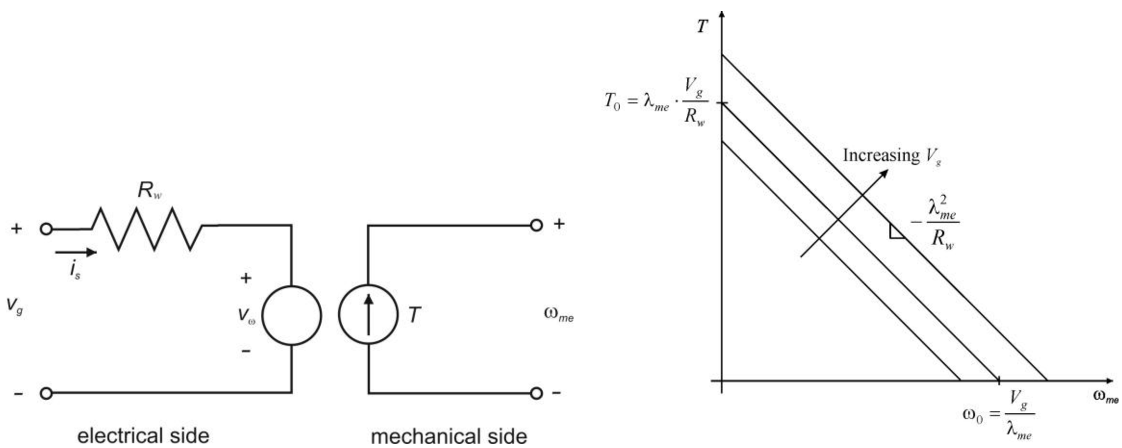



*Fig. 1. Quasistatic model for the PMS motor (a) and quasistatic torque (T) versus speed ($\omega_{me}$) with $V_g$ as a parameter (b). If the speed-torque point at a given current is above the torque values on the plot for the corresponding Vg, then Vg must be increased.*

The current-control loop increases duty-ratio $D$ and average per-cycle PWM terminal voltage $D \cdot V_g$ increases. As the motor torque line moves away from the origin, its intersection with the commanded torque line will occur at a higher speed, and speed will increase toward the new intersection value.

For step-motors, $\lambda_{me}$ (the electromechanical flux of the motor) is larger than for PMS motors having few pole-pairs. To ilustrate, for size 23 and 34 step-motors of 50 pole-pairs, each step is typically 1.8° mechanical (me) per ¼ electrical (el) cycle or "full step";

$$90 \text{ deg el}/50 = 1.8 \text{ deg me , full step}$$

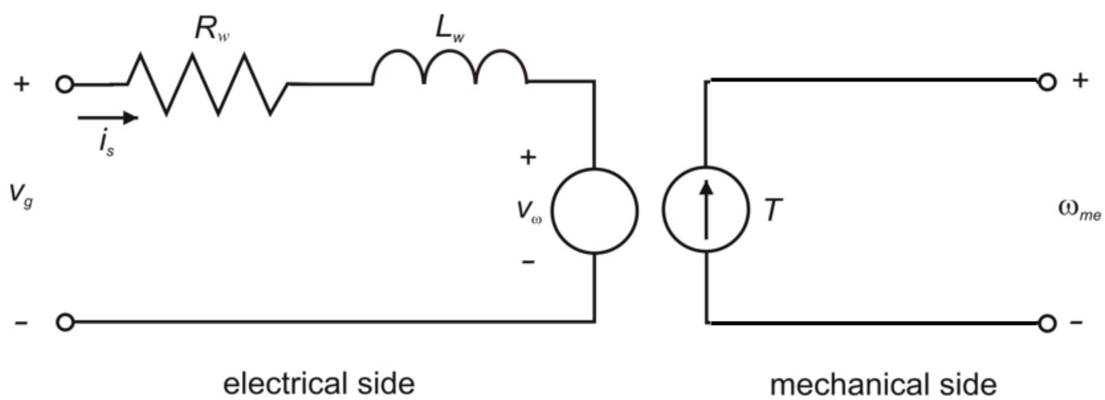
A typical size 23 step-motor is a Lin 5718L-01P which has pole-pairs $p$ = 50, $\lambda_{me}$ = 0.525 V·s ≈ 3.3 V/Hz me, $R_w$ = 1.1 Ω, $L_w$ = 3.9 mH, and $I_{max} = \sqrt{2} \cdot 2.8$ A = 3.96 A ≈ 4 A (current magnitude for both phase-windings). For this $\lambda_{me}$ and $\omega_0$ = 3 krpm = 50 Hz me, the required

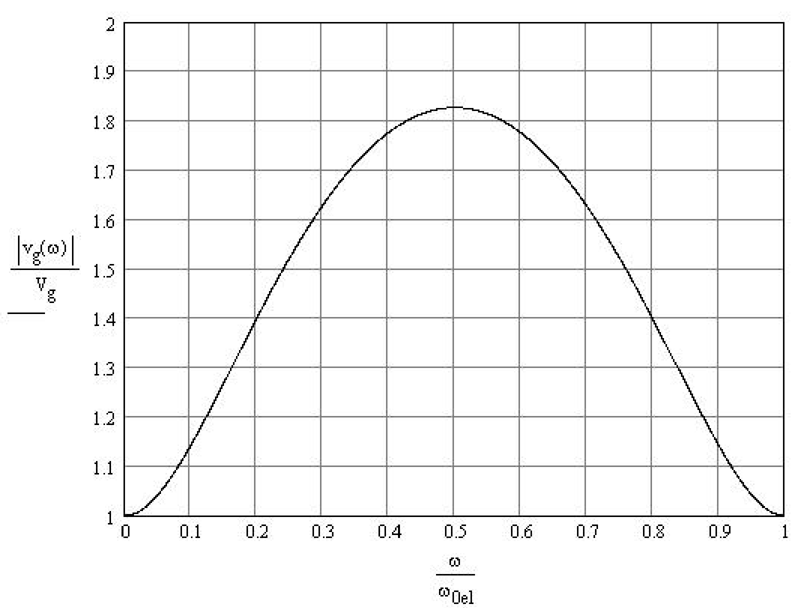
$$V_g = \lambda_{me} \cdot \omega_0 = (3.3\ \text{V/Hz me}) \cdot (50\ \text{Hz me}) = 165\ \text{V}$$

At the nominal 24 V found in the motor specification, the maximum motor speed is only 7.27 Hz me ≈ 436 rpm. Higher speeds for quicker move times require higher $V_g$. This rather large voltage at $\omega_0$ compared to 24 V is needed because motor behavior is constrained to follow the motor torque-speed line $T(\omega_{me})$. At stall ($\omega_{me}$ = 0 Hz), motor current is $I_0$ = 21.8 A, well above the maximum specified current of 4 A.

Even more voltage is needed to overcome the drop across $Z_s$, and especially across $j \cdot \omega_{el} \cdot L_w$, which increases with frequency. Fig. 2a repeats the dynamic model, showing the winding impedance $Z_w$. The graph of Fig. 2b plots drive voltage (relative to $V_g$) as a function of motor speed (relative to no-load speed). It takes both induced and inductive voltages into account for unconstrained current. $V_g$ must be over 1.8 times its quasistatic value to achieve the quasistatic current at mid-speed.




(a)




(b)

*Fig. 2. Dynamic field-oriented PMS motor model with winding impedance $Z_w$: winding resistance $R_w$ in series with winding inductance $L_w$ (a). $V_g$ multiplier as a function of motor speed $\omega_{me}$ normalized to the no-load speed (b). Maximum drive voltage is required to follow the quasistatic $T(\omega_{me})$ line at half the no-load speed.*

Winding inductance forms a series RL impedance of

$$Z_w = R_w + j \cdot \omega_{el} \cdot L_w$$

The voltage dropped across $Z_w$ is in series with $v_\omega$. If the phase of $v_g$ leads $v_\omega$ by the amount of phase shift caused by $Z_w \cdot i_s$ then $i_s$ can align with $v_\omega$ for field-oriented drive. The field-oriented phase of torque angle $\delta = \pi/2 = 90°$ and is the angle between the stator current $i_s$ and the stator flux $\lambda_s$ which lags behind $v_\omega = d\lambda_s/dt$ by $\theta = \pi/2$. Thus, field-oriented phase aligns $i_s$ with $v_\omega$ so that their power product is maximum. As vectors,

$$P = \mathbf{i}_s \bullet \mathbf{v}_\omega = i_s \cdot v_\omega \cdot \cos\theta_s = i_s \cdot v_\omega \cdot \sin\delta$$

where $\theta_s$ is the angle between $i_s$ and $v_\omega$.

The required phase lead is maximum at mid-scale—at mid-speed of $\omega_0/2$ and mid-torque of $T_0/2$. With current-source winding drive, $i_s$ is aligned with $v_\omega$ but phase sensing of $v_\omega$ across the motor terminals is no longer direct because of phase error caused by $Z_w$. Winding reactance phase compensation will appear again in a later article.

If $v_g$ is not increased but kept constant at $v_g = V_g$, then current is less than for the quasistatic motor and $T(\omega_{me})$ sags, as shown by the dotted curve in Fig. 3. This concave curve shape is common for $T(\omega_{me})$ curves in motor catalog data.

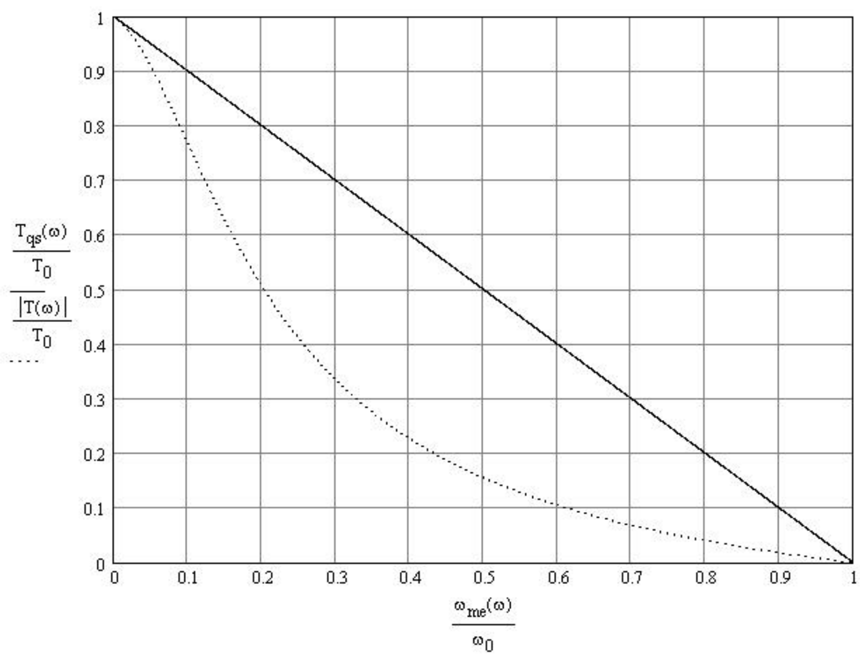



*Fig. 3. Quasistatic torque (solid plot) and torque (dotted plot) caused by inductive reactance.*

When total current magnitude $\leq I_{max} = \sqrt{2} \cdot 2.8\text{ A} \approx 4\text{ A}$, then the Fig. 4 plots result with an increase of $V_g$ to $V_g = 175$ V. Over most of the speed range, $i_s = I_{max}$ and decreases only when $\omega_{me}$ draws near to the last 4% of $\omega_0$ at the high end of the speed range. This shows how much more current the windings are capable of relative to maximum allowable motor power dissipation and suggests current-source drive whereby current is under the direct control of the motor-drive. This also makes rotor phase from $v_\omega(t)$ possible to sense across the high impedance of the drive source.

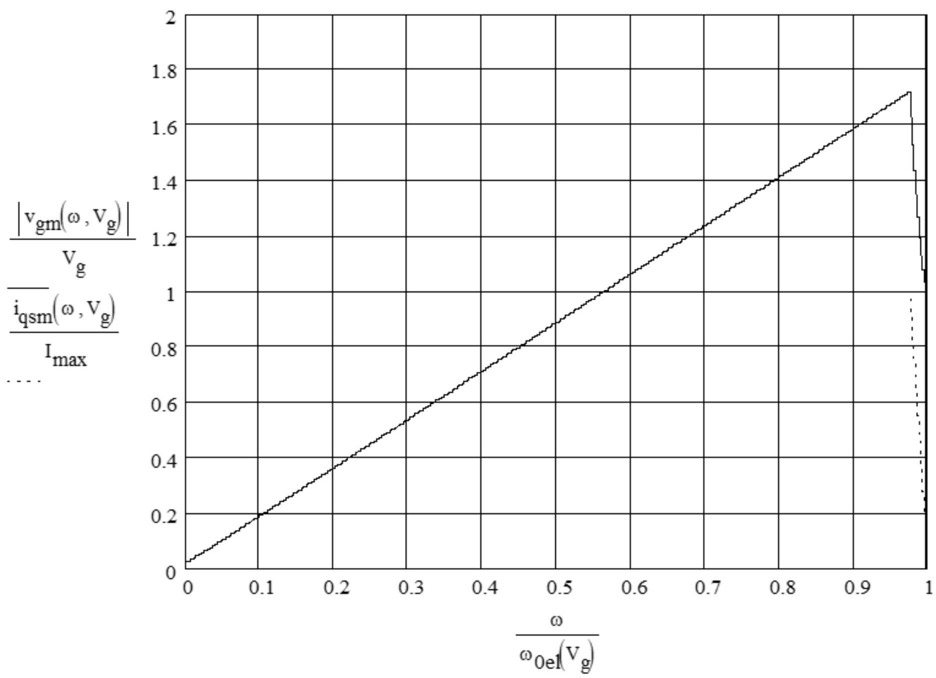



*Fig. 4. Increase of stator terminal voltage (solid plot) to maintain quasistatic conditions of torque with speed. The corresponding current (dotted plot) is at $I_{max}$ (1 on plot) over most of $\omega_{me}$ then decreases when the $I_{max}$ limit is reached.*

The current corresponding to the quasistatic torque (Fig. 4 dotted horizontal line at 1, buried in graticule) is constant at $I_{\max}$ until it reaches

$$\omega_{\max} = \frac{V_g - R_w \cdot I_{\max}}{\lambda_{me}} \approx 51.73 \text{ Hz me}$$

For $V_g$ = 175 V, at stall the maximum $V_g$ must be about 1.72 times higher, or about (1.72)·(175 V) ≈ 301 V.

The slope of $v_g(\omega_{me})$ from the Fig. 4 graph is, taken from $\omega/\omega_{0el}$ = 0.115 to 0.915:

$$\frac{\Delta v_g}{\Delta \omega_{me}} = \frac{\Delta v_g}{\Delta \omega} \cdot \left( p \cdot \frac{\text{Hz el}}{\text{Hz me}} \right) \approx \frac{(1.4) \cdot (175 \text{ V})}{(0.8) \cdot (2653 \text{ Hz el})} \cdot \left( 50 \cdot \frac{\text{Hz el}}{\text{Hz me}} \right) = 5.77 \text{ V/Hz me}$$

where $V_g$ = 175 V and $\omega_{0el}$ = 2653 Hz el. This slope has two components: the induced voltage $\lambda_{me} \cdot \omega_{me}$ and the voltage drop across the series winding inductance $[p \cdot \omega_{me}] \cdot (j \cdot L_w \cdot I_{\max})$. When they are added as complex numbers and their magnitude taken,

$$\frac{dv_g}{d\omega_{me}} = \sqrt{\lambda_{me}^2 + [p \cdot L_w \cdot I_{\max}]^2} = \sqrt{(0.535 \text{ V} \cdot \text{s})^2 + (50 \cdot (3.8 \text{ mH}) \cdot (3.96 \text{ A}))^2}$$

$$= (0.918 \text{ V} \cdot \text{s}) \cdot \frac{2\pi \text{ s}^{-1}}{\text{Hz me}} = 5.77 \text{ V/Hz me}$$

$I_{max}$ cannot be sustained above $\omega_{max}$ without increasing $V_g$ even more. Thus $\omega_{max}$ is the specified maximum speed of the motor-drive capable of sustaining the maximum torque produced by $I_{max}$, or $\omega_{max} \geq 3$ krpm = 50 Hz me. For these parameters, $\omega_0$ = 53.05 Hz me. The resulting torque plot of Fig. 5 shows the $I_{max}$-limited torque with a fixed 175-V motor supply. The torque sag is compensated by a boosted $V_g$ so that the current will follow the quasistatic (solid-line) plot, shown above in Fig. 3. This corresponds to the dashed line of $T_{qsm}$ on the torque plot of Fig. 5.

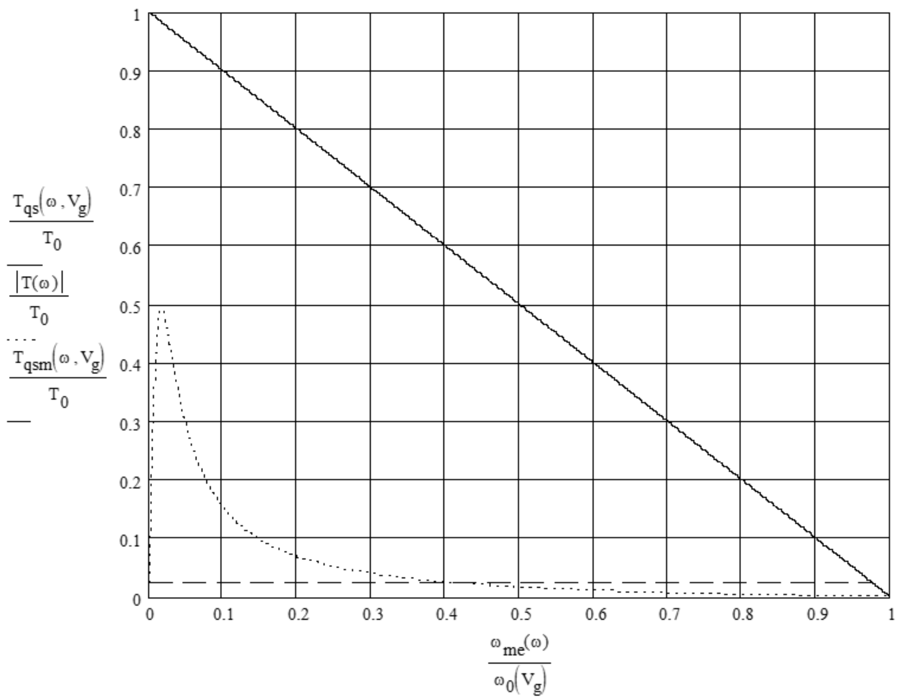



*Fig. 5. Plots of quasistatic torque (solid), $I_{max}$-limited torque (dash), and dynamic torque (dot) with speed. All variables are normalized to their maximum values of $T_0$ for torque and $\omega_0$ for $\omega_{me}$.*

The maximum required voltage is the current-constrained $v_g(\omega_{max}) = v_{gmax}$. It is found by substituting the expression for $\omega_{max}$ into

$$v_g(\omega_{me}) = Z_s(\omega_{me}) \cdot I_{max} + v_\omega(\omega_{me})$$

The result is a complex number. Its magnitude is the value of maximum $V_g$ required, or

$$v_{g\max} = V_g + j \cdot \left( \omega_0 - \frac{R_w \cdot I_{\max}}{\lambda_{me}} \right) \cdot p \cdot L_w \cdot I_{\max}$$

Then

$$|\, v_{g\max} \,| = \sqrt{V_g^2 + \left( \omega_0 - \frac{R_w \cdot I_{\max}}{\lambda_{me}} \cdot p \cdot L_w \cdot I_{\max} \right)^2}$$

The basic limits on $T(\omega_{me})$ are the motor maximum current, speed, and power limits. As $V_g$ increases, both $\omega_0$ and $T_0$ increase. Unconstrained power at $V_g$ = 175 V is nearly 7 kW, well beyond what a size-23 motor could convert. With current drive, instead of occurring at mid-speed, maximum power occurs at maximum speed and $I_{max}$, or at about 53 Hz me, just above the maximum specified speed.

Calculated from the electrical side, this is about (3.96 A)·(175 V) = 693 W. From the mechanical side, $T_{max}$ = 2.08 N·m at a speed of 53 Hz me results in mechanical power of about 693 W. This is well beyond the roughly 150-W continuous limit of a size-23 step-motor. This peak power is required to achieve acceleration at $I_{max}$ over the entire speed range, and if the acceleration times are limited, the average motor power can still be within its rating.

With field-oriented control, it is possible to achieve these higher performance values. To achieve them requires higher-performance circuitry. The driver must sustain about 300 V and 3 A for a short duration during acceleration to fully utilize the motor capability. At the least-demanding end of the performance range, the motor-drive must also be able to supply voltage well below the minimum quasistatic $V_g$ ( = 175 V) for low-torque, low-speed moves.

A switching supply with a $D/D'$ transfer function will provide voltage both higher and lower than a 24-V input (or a 12-V to 48-V input range). A Ćuk-derived converter is capable of this and a SEPIC with minimized input current ripple is the most attractive alternative for non-inverting, non-isolated conversion for small step-motors. If on-times are synchronized with the motor-driver PWM, then overall motor current ripple is minimized.

A quasistatically-controlled $V_g$ can reduce motor power dissipation and ripple current. If $V_g$ is set to approximately twice the required motor voltage, then the PWM duty ratio will remain around 50%. At $D$ = 0.5, current ripple is minimum. (It is maximum at $D$ = 0+ and 1−.)

## *Design For Maximum Efficiency*

Loci of constant power on the torque-speed graph of Fig. 5 would be hyperbolas of constant $T \cdot \omega_{me}$. Maximum power for a given $V_g$ occurs at *mid-speed* $\omega_0/2$ and *mid-torque* $T_0/2$ in speed and torque ranges. On the electrical and mechanical sides of the motor, this corresponds to

$$P_{max} = \frac{(V_g/2)^2}{R_w} = \frac{V_g}{2} \cdot \frac{I_0}{2} = \frac{T_0}{2} \cdot \frac{\omega_0}{2} = \frac{\hat{P}_{el}}{4} = \frac{\hat{P}_{me}}{4}$$

Another criterion for adjusting $V_g$ is to operate the motor at maximum efficiency. Mechanical output power is

$$P_{me} = T \cdot \omega_{me} = \left( -\frac{\lambda_{me}^2}{R_w} \cdot \omega_{me} + \lambda_{me} \cdot \frac{V_g}{R_w} \right) \cdot \omega_{me} = \frac{\lambda_{me} \cdot \omega_{me}}{R_w} \cdot \left( V_g - \lambda_{me} \cdot \omega_{me} \right)$$

Electrical input power is

$$P_{el} = \dot{W}_{el} = V_g \cdot i_s = V_g \cdot \frac{V_g - \lambda_{me} \cdot \omega_{me}}{R_w} = I_0 \cdot (V_g - \lambda_{me} \cdot \omega_{me})$$

Efficiency of output mechanical power to input electrical power in the ideal case with no core or motor-drive losses reduces to

$$\eta(\omega_{me}) = \frac{P_{me}}{P_{el}} = \frac{\omega_{me}}{\omega_0}$$

The plots, normalized to maximum mechanical and electrical power, are shown in Fig. 6. Based on this simplified efficiency model, maximum motor power occurs at mid-speed where efficiency is 50%. Maximum efficiency is attained at no-load speed, where electrical loss is zero because current is zero. Neither is any torque produced there.

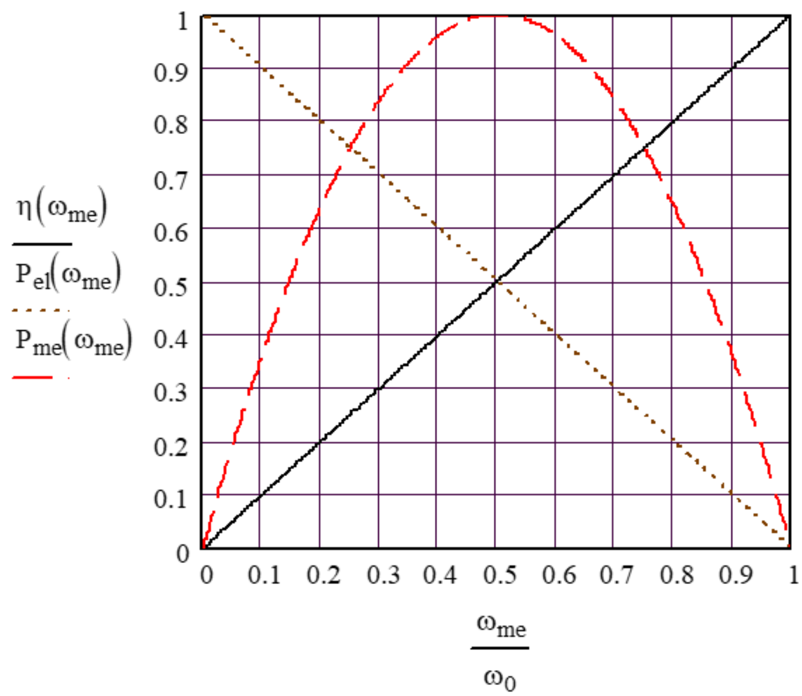



*Fig. 6. Plots of motor efficiency (solid), and electrical (dot) and mechanical (dash) power, with speed.*

This model only includes the loss across the winding resistance, the motor electrical loss. A more accurate model includes the magnetic loss. This loss is a complicated function of motor speed, but somewhat greater accuracy can be achieved by including just the hysteresis component of magnetic loss, which is linear with speed. It is added in the motor circuit model as shunt resistance $R_m$ across $v_\omega$. Then

$$P_{mem} = V_g \cdot \frac{V_g - \lambda_{me} \cdot \omega_{me}}{R_w} - \frac{(\lambda_{me} \cdot \omega_{me})^2}{R_m}$$

where the mechanical power is the electrical power minus the hysteresis loss, the second term in the above expression. Then the efficiency is

$$\eta_m(\omega_{me}) = \frac{\omega_{me}}{\omega_0} \cdot \left(1 - \frac{R}{R_m} \cdot \frac{\omega_{me}}{\omega_0 - \omega_{me}}\right)$$

This efficiency, with magnetic hysteresis loss, is plotted in Fig. 7 for values of $R_m = a \cdot R_w$, for $a$ = 1, 10, and 100.

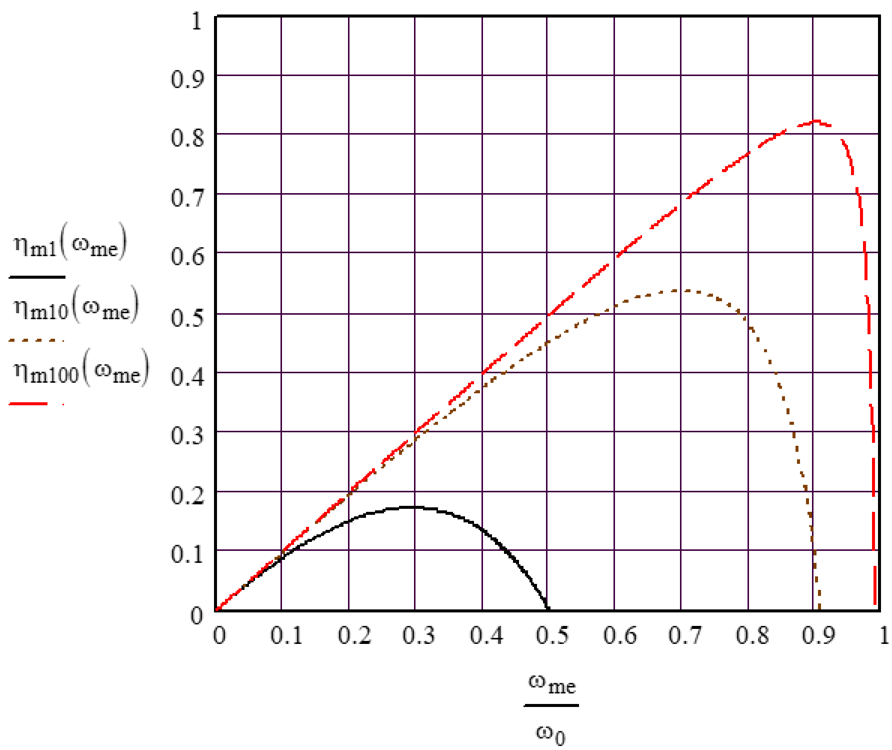



*Fig. 7. Efficiency plots with magnetic hysteresis loss for $R_m/R_w = a = 1$ (solid), 10 (dot), and 100 (dash).*

Based on this model, a motor with dominant winding loss relative to magnetic hysteresis loss ($a >> 1$) will achieve maximum efficiency near no-load speed. Because $I_{max} << I_0$, adjusting $V_g$ to deliver $I_{max}$ will always operate the motor near the maximum-efficiency speed.

Optimally the motor is not operated anywhere near its maximum power at mid-speed; it is operated in the region of maximum efficiency, near its no-load speed. This is like operating a linear power amplifier with an adjustable power supply that varies its voltage with the amplified waveform to keep the power loss across the amplifier output stage minimal. Thus a compromise between an operating speed at maximum power as opposed to maximum efficiency is a design consideration.

**References**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5. "Motor Control For Designers (Part 5): Deriving Force Production From Magnetic Energy" by Dennis Feucht, Innovatia Laboratories, How2Power Today, January 2026.
6. "Motor Control For Designers (Part 6): Three-Phase Motor Waveforms" by Dennis Feucht, Innovatia Laboratories, How2Power Today, April 2026.


 

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## About The Author

*Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.*

For further reading on motors and motor drives, see "A Practical Primer On Motor Drives".