

Hard Ferrite Inductor Core Analytical Model And Its Paradoxical Property

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A previous article analyzed the relationship between inductance and operating current, developing an analytical model for inductance that could be used to predict saturation in inductors.^[1] That model was specifically relevant for power supply applications where soft ferrite cores are used. However, there are also many non-power supply applications employing hard ferrites, where it is beneficial to model the inductance versus current relationship.

For example, in motors, generators, electromagnetic relays and speakers, hard ferrite cores are used. In these cases, an unusual property of hard ferrites—the ability to "memorize" a previous magnetic status of the core—is a factor: a peak or hump is displaced in the permeability curve versus where it appears for soft ferrites at $H_x = 0$ A/m. This occurs because the demagnetizing field has to remove the core magnetization first and then magnetize the core in an opposite direction. In this article, we develop a model for inductance in hard ferrite-based inductors that accounts for this property.

Accounting For Core Remanence In B-H Expressions

An in the earlier analysis of current influence on inductance,^[1] we begin with analytical expressions obtained previously^[2] that describe a magnetic core hysteresis loop using just inductor-plausible data sheet parameters.

$$B_1(H_x) = B_{sat} \left[\frac{2}{1 + e^{\frac{-(H_x - H_c)}{H_{01}}}} - 1 \right] \quad (1)$$

$$B_2(H_x) = B_{sat} \left[\frac{2}{1 + e^{\frac{-(H_x + H_c)}{H_{02}}}} - 1 \right] \quad (2)$$

Here B_{sat} is the core saturation flux density; H_c is the core coercive force; and H_{01} and H_{02} are virtual magnetic field strength for each hysteresis branch respectively, the expressions for which will be determined later. H_x is the present magnetic field strength. $B_1(H_x)$ and $B_2(H_x)$ are values for present flux densities for both branches of the hysteresis loop. The B_{sat} and H_c terms are taken from the cores' data sheet parameters.

In many cases, such as for hard ferrites, it is crucial to know the core remanence B_{r1} and B_{r2} for the two branches of the hysteresis loop. Formulas for remanence may be obtained from equations (1) and (2) by equating $H_x = 0$ as shown below.

$$B_{r1} = B_{sat} \left[\frac{2}{1 + e^{\frac{-(0 - H_c)}{H_{01}}}} - 1 \right] \quad (3)$$

$$B_{r2} = B_{sat} \left[\frac{2}{1 + e^{\frac{-(0 + H_c)}{H_{02}}}} - 1 \right] \quad (4)$$

Since B_{r1} , B_{r2} , B_{sat} and H_c are magnetic core data sheet values, formulas (3) and (4) provide solutions for H_{01} and H_{02} .

$$H_{01} = \frac{H_c}{\ln\left(\frac{1}{\frac{B_{r1}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)} \quad (5)$$

$$H_{02} = \frac{H_c}{\ln\left(\frac{1}{\frac{B_{r2}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)} \quad (6)$$

Therefore, plugging in (5) and (6) into (1) and (2) respectively, we can obtain expressions for both branches of the hysteresis loop:

$$B_1(H_x) = B_{sat} \left[\frac{2}{\frac{-(H_x - H_c)}{H_c} \cdot \frac{1}{\ln\left(\frac{1}{\frac{B_{r1}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)}} - 1 \right] \quad (7)$$

$$B_2(H_x) = B_{sat} \left[\frac{2}{\frac{-(H_x + H_c)}{H_c} \cdot \frac{1}{\ln\left(\frac{1}{\frac{B_{r2}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)}} - 1 \right] \quad (8)$$

The median value of the hysteresis loop can be expressed as

$$B_0(H_x) = \frac{B_{sat} \left[\frac{2}{\frac{-(H_x - H_c)}{H_c} \cdot \frac{1}{\ln\left(\frac{1}{\frac{B_{r1}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)}} - 1 \right] + B_{sat} \left[\frac{2}{\frac{-(H_x + H_c)}{H_c} \cdot \frac{1}{\ln\left(\frac{1}{\frac{B_{r2}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)}} - 1 \right]}{2}$$

After simplification we get

$$B_0(H_x) = - \frac{B_{sat} \left[\left(\frac{1}{\frac{B_{r1}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1 \right)^{\frac{H_c - H_x}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_x}{H_c}} - 1 \right]}{\left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_x}{H_c}} + 1 \right] \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_x}{H_c}} + 1 \right]} \quad (9)$$

Assuming the magnetic core permeability is

$$\mu(H_x) = \frac{d}{dH_x} B_0(H_x) \quad (10)$$

we obtain

$$\mu(H_X) =$$

$$\begin{aligned} & \frac{B_{sat} \left[\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} - \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} - 2 \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{B_{sat} \left[-\ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{2(H_c + H_X)}{H_c}} + 2 \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{(H_c + H_X)}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{B_{sat} \left[+\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{2(H_c + H_X)}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \end{aligned} \quad (11)$$

which gives an initial magnetic core permeability at $H_x = 0$.

$$\mu_{ini} = \frac{B_{sat} \cdot \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)}{4 \cdot H_c} - \frac{B_{sat} \cdot \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)}{4 \cdot H_c} - \frac{B_{r1}^2 \cdot \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)}{4 \cdot B_{sat} \cdot H_c} + \frac{B_{r2}^2 \cdot \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)}{4 \cdot B_{sat} \cdot H_c} \quad (12)$$

Using a conventional formula for inductance we get

$$L_m(H_X) = \frac{S_m \cdot N_W^2}{l_m} \cdot \mu(H_X) \quad (13)$$

Then we get

$$L_m(H_X) =$$

$$\begin{aligned} & \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} - \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} - 2 \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[-\ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{2(H_c + H_X)}{H_c}} + 2 \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{(H_c + H_X)}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \\ & + \end{aligned}$$

$$\frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[+ \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{2(H_c + H_X)}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \quad (14)$$

Noting that H_X can be expressed in terms of magnetizing current I_m ,

$$H_X = \frac{I_m \cdot N_W}{l_m}$$

We can substitute for H_X in equation (14) to obtain the following expression for inductance in terms of current.

$$L_m(I_m) =$$

$$\begin{aligned} & \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c + I_m \cdot N_W}{H_c}} - \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} - 2 \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + I_m \cdot N_W}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + I_m \cdot N_W}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[- \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{2(H_c + I_m \cdot N_W)}{H_c}} + 2 \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{(H_c + I_m \cdot N_W)}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + I_m \cdot N_W}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[+ \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{2(H_c + I_m \cdot N_W)}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + I_m \cdot N_W}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - I_m \cdot N_W}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + I_m \cdot N_W}{H_c}} + 1 \right]^2} \end{aligned} \quad (15)$$

Design Example And Result Analysis

Let's define the dependence of an inductor vs. current, having parameters as follows:

μ_r = magnetic core material relative permeability = 1510

S_m = magnetic core cross-sectional area (m²) = 31 mm²

μ_{ini} = initial permeability of the magnetic core = $\mu_0 \cdot \mu_r$

l_m = magnetic line length of the magnetic core = 47 mm

H_c = coercive force of the magnetic core = 160 A/m

B_{sat} = permissible magnetic flux density saturation value in the core = 0.45 T

N_w = number of winding turns = 44

B_{r1} = core remanence, specific for hard ferrites = -0.42 T

B_{r2} = core remanence, specific for hard ferrites = 0.42 T

The first five magnetic core parameters are taken from the manufacturer's data sheet.

$$\mu_{ini} = \frac{B_{sat} \cdot \ln\left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}}\right)}{4 \cdot H_c} - \frac{B_{sat} \cdot \ln\left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}}\right)}{4 \cdot H_c} - \frac{B_{r1}^2 \cdot \ln\left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}}\right)}{4 \cdot B_{sat} \cdot H_c} + \frac{B_{r2}^2 \cdot \ln\left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}}\right)}{4 \cdot B_{sat} \cdot H_c}$$

$$= 6.103 \times 10^{-4} \frac{\text{kg} \cdot \text{m}}{\text{A}^2 \cdot \text{s}^2}$$

$$\mu_r = \frac{\mu_{ini}}{\mu_0} = 485.679$$

To build the hysteresis loop, recalling equations (7), (8) and (9), we have

$$B_1(H_x) = B_{sat} \left[\frac{\frac{2}{\frac{-(H_x - H_c)}{H_c}} - 1}{1 + e^{\ln\left(\frac{1}{\frac{B_{r1}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)}} \right]$$

$$B_2(H_x) = B_{sat} \left[\frac{\frac{2}{\frac{-(H_x + H_c)}{H_c}} - 1}{1 + e^{\ln\left(\frac{1}{\frac{B_{r2}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1\right)}} \right]$$

$$B_0(H_x) = - \frac{B_{sat} \left[\left(\frac{1}{\frac{B_{r1}}{2 \cdot B_{sat}} + \frac{1}{2}} - 1 \right)^{\frac{H_c - H_x}{H_c}} \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_x}{H_c}} - 1 \right]}{\left[\left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_x}{H_c}} + 1 \right] \cdot \left[\left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_x}{H_c}} + 1 \right]}$$

The median value $B_0(H_x)$ has an initial value for μ_{ini} that is lower than the permeability of adjacent areas. The three expressions above are plotted for the design example in Fig. 1.

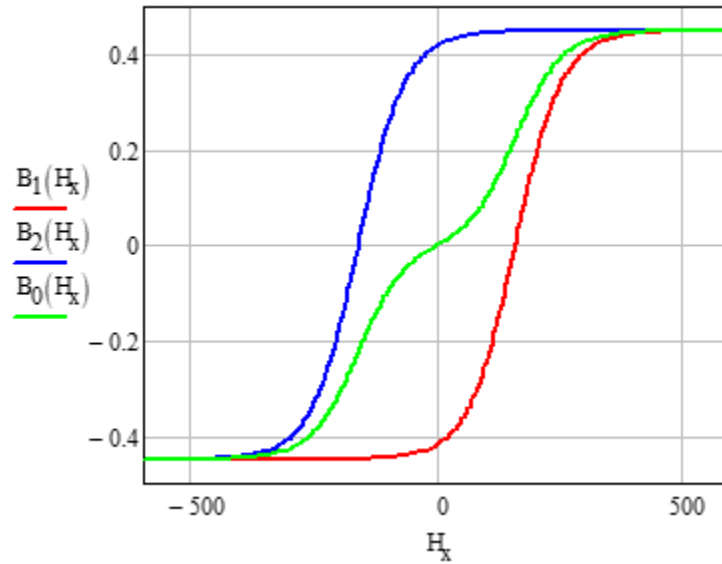


Fig. 1. A hysteresis loop for a hard ferrite core. The green trace is for the median characteristic.

Then, from equation (11) we have

$$\mu(H_X) =$$

$$\begin{aligned} & \frac{B_{sat} \left[\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} - \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} - 2 \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{B_{sat} \left[-\ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{2(H_c + H_X)}{H_c}} + 2 \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{(H_c + H_X)}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \\ & + \\ & \frac{B_{sat} \left[+\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{2(H_c + H_X)}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - H_X}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + H_X}{H_c}} + 1 \right]^2} \end{aligned}$$

which leads to the plot in Fig. 2.

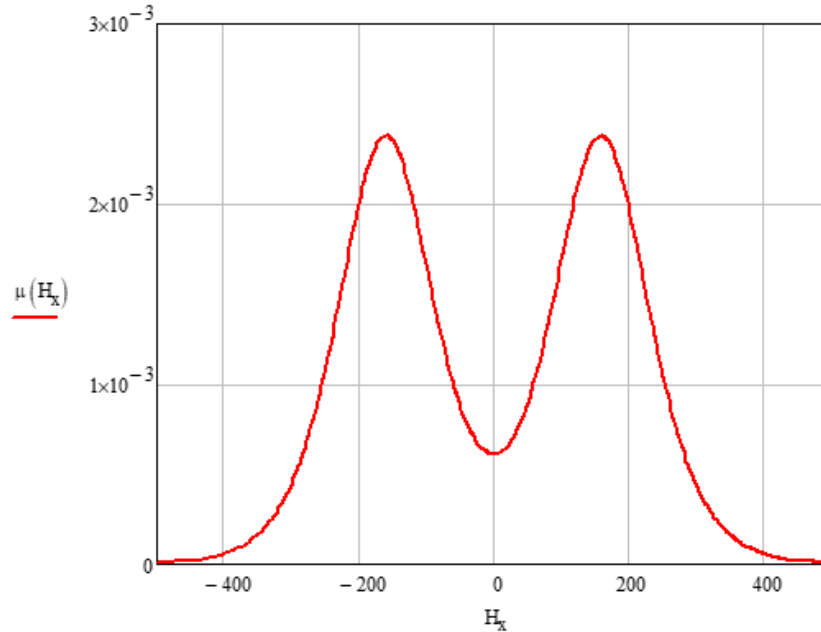


Fig. 2. Permeability initial value is not maximum.

Finally, we can apply equation (15) to obtain a plot of inductance as a function of current as shown in Fig. 3.

$$L_m(H_x) =$$

$$\frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + \frac{I_m \cdot N_W}{l_m}}{H_c}} - \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} - 2 \ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} \cdot \left(-\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + \frac{I_m \cdot N_W}{l_m}}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + \frac{I_m \cdot N_W}{l_m}}{H_c}} + 1 \right]^2}$$

$$+ \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[-\ln \left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{2(H_c + \frac{I_m \cdot N_W}{l_m})}{H_c}} + 2 \ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{(H_c + \frac{I_m \cdot N_W}{l_m})}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + \frac{I_m \cdot N_W}{l_m}}{H_c}} + 1 \right]^2}$$

$$+ \frac{S_m \cdot N_W^2}{l_m} \cdot \frac{B_{sat} \left[\ln \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right) \cdot \left(-\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{2(H_c + \frac{I_m \cdot N_W}{l_m})}{H_c}} \cdot \left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + \frac{I_m \cdot N_W}{l_m}}{H_c}} \right]}{H_c \left[\left(\frac{B_{r1} - B_{sat}}{B_{r1} + B_{sat}} \right)^{\frac{H_c - \frac{I_m \cdot N_W}{l_m}}{H_c}} + 1 \right]^2 \cdot \left[\left(\frac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}} \right)^{\frac{H_c + \frac{I_m \cdot N_W}{l_m}}{H_c}} + 1 \right]^2}$$

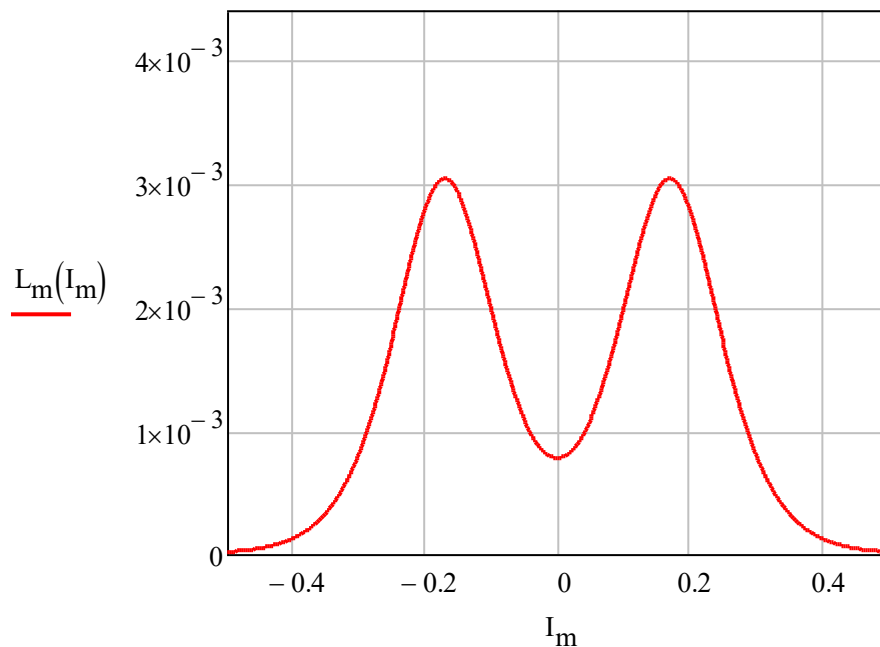


Fig. 3. Inductance has a minimum at zero current. Inductance is plotted here in henries (H) versus amps (A).

The Takeaway

The inductance vs. current dependence for a hard ferrite has a hump placed away from $H_x = 0$ due to the high remanence of the core material. This is in stark contrast to the L_m vs. I_m curve for soft ferrites, which peaks at $H_x = 0$.

Therefore, during operation of a hard ferrite core, the magnetic domains that have some initial magnetization due to the “magnetic hardness” of the core need to be demagnetized in the opposite direction first to reach zero residual magnetization. That state corresponds to the maximum permeability or inductance not at zero external field H_x (as in soft ferrites) but at some field that demagnetizes the initial magnetization.

Note that the shifts of the characteristic hump away from the $H_x = 0$ ordinate occur in all cores. This is true for any remanence value. But for soft ferrites the remanence is so low that the hump displacement is unnoticeable.

References

1. [“Inductor Analytical Model And Its Inductance Dependence On Current”](#) by Gregory Mirsky, How2Power Today, May 2026.
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About The Author



Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an

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