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Using Feedback Loop Properties To Optimize Transient Response Independently Of Converter Topology

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In a power converter, the small-signal output voltage deviation ΔV(s) in response to load current step ΔI is given by

$$\Delta V(s) = -\frac{\Delta I}{s} \times Z_{CL}(s) \qquad (1)$$

where $Z_{CL}(s)$ is the closed-loop output impedance.

It is known that feedback reduces the output impedance by a factor of 1+T(s), where T(s) is the converter's open loop transfer function.[1] If the original open-loop output impedance was $Z_{OL}(s)$, then the closed loop impedance is given as

$$Z_{CL}(s) = \frac{Z_{OL}(s)}{1+T(s)} \qquad (2)$$

Since inductors can't follow rapid current changes, the transient current flows primarily through output capacitance. Therefore, for step load transient analysis we can assume that the open-loop output impedance is dominated by capacitance C with its ESR. Then

$$\Delta V(s) \approx -\frac{\Delta I}{s} \times \frac{Rc+\frac{1}{sC}}{1+T(s)} \qquad (3)$$

For a known T(s) one can solve (3) for the step-load response by taking the inverse Laplace transform. It's possible to construct a complete plant–modulator-compensator gain T(s) of a converter and derive an equation predicting transient response. However, this will yield a high-order transfer function which makes symbolic solutions unwieldy and obscures the direct influence of key design parameters.

Besides this, the result will be based on specific converter topologies or control method, which limits its portability. The good news is that for transient analysis we don't need to know the complete transfer function of the converter.

It's known that phase margin φ_M and consequently the transient response are determined by the behavior of the loop gain T(s) *near the crossover frequency* ω_C. Specifically, these properties are related to the local slope of T(s) magnitude at ω_C.[1,3] In engineering practice, for the transient analysis it is common to approximate the open-loop transfer function near crossover by a second-order form with a single pole at the origin and another high-frequency pole above crossover.[1,2]

This approximation produces a second-order function which is a cornerstone of classical control theory and is well suited for *input step* analysis which deals with T/(1+T). However, when applied to *load step* analysis which deals with 1/(1+T) according to (3), it yields a steady-state dc voltage offset and distortion of the predicted waveform because $Z_{CL}(s)$ does not vanish as $s \to 0$.

In this article, we construct a loop gain formulation that enforces zero dc output impedance for $s \to 0$ while yielding a compact and analytically tractable expression for the transient response in terms of bandwidth, phase margin and output capacitance.

The derived equation for load transient response is demonstrated with a set of example conditions at different values of phase margin. These analytical results are then verified with an LTspice simulation. As we'll see, the expression obtained here offers a means of tuning the transient response.

As the discussion continues, the expression for load transient response is used to determine expressions for peak voltage, settling time and overshoot. In particular, this allows us to observe the relationship between setting time and phase margin, which is demonstrated using the example conditions.

Construction Of Generic Open-Loop Transfer Function

Crossover frequency (also called bandwidth) ω_C is defined as the frequency at which the magnitude $|T(j\omega)|$ equals unity (or 0 dB in logarithmic scale). Phase margin is the difference between the argument of $T(j\omega)$ at the crossover frequency and -180°:

$$\varphi_M = \angle T(j\omega_C) + 180^\circ \qquad (4)$$

Well-stabilized voltage converters free of right-half plane singularities and abrupt resonance near the crossover are characterized by *minimum-phase* behavior. It follows from the Hilbert transform that for such systems phase margin is uniquely related to the local slope of the gain's magnitude at crossover.[3,4] To analyze the transient response, we aim to derive a reverse relationship: to reconstruct a T(s) which results in a particular phase margin at a given crossover frequency.

Let's consider two boundary cases:

- $T(s) = \omega_C/s$ on log-log scale has slope $m = -1$ (20 dB/decade) yielding phase margin $\varphi_M = 90^o$ (stable condition).
- $T(s) = (\omega_C/s)^2$ has slope $m = -2$ (40 dB/decade) yielding phase margin $\varphi_M = 0^o$ (oscillatory-like).

Between these two boundaries for a stabilized converter the T(s) near ω_C can be described as a fractional-order function:

$$T(s) = \left(\frac{\omega_C}{s}\right)^n \qquad (5)$$

where $1 < n < 2$.

By substituting $s = j \times \omega_C$ in (5) we find the phase of this function at crossover:

$$\angle T(j\omega_C) = -n \times \frac{\pi}{2} = -n \times 90^o \qquad (6)$$

Then from (4) phase margin is

$$\varphi_M = 180^o - n \times 90^o \qquad (7)$$

Rearranging (7) we express the power "n" of function (5) in terms of phase margin:

$$n = 2 - \frac{\varphi_M}{90^o} \qquad (8)$$

Substituting (8) into (5) yields a generic relationship between loop gain near crossover frequency and phase margin:

$$T(s) = \left(\frac{\omega_C}{s}\right)^{\left(2 - \frac{\varphi_M}{90^o}\right)} \qquad (9)$$

Accordingly, the slope on a log-log scale of the function (5) at crossover is

$$m = -n = \frac{\varphi_M}{90^o} - 2 \qquad (10)$$

Theoretically, by substituting (9) into (3) and taking the inverse Laplace transformation, one can find a generic step load transient response in terms of bandwidth and phase margin. However, with the fractional-order model (9) the result yields a Mittag-Leffler function which is difficult to deal with and which masks the direct influence of key design parameters.

To avoid this complexity while preserving the essential loop behavior, we will construct a dynamically equivalent function T(s) that behaves similarly to (9) around the crossover frequency. Namely, we need a T(s) that satisfies three invariants at ω_C:

- Unity gain $|T(j\omega_C)| = 1$;
- Phase $\angle T(j\omega_C) = \varphi_M - 180^o$ according to (4);
- Magnitude's slope "m" approximating (10).

Let us examine the following function:

$$T(s) = \frac{\omega_C}{s} \times \frac{\sin\varphi_M \times s + \cos\varphi_M \times \omega_C}{s} \qquad (11)$$

The first term in (11) represents slope m = –1 corresponding to $\varphi_M = 90^o$. The second one is a factor that "shapes" the slope in terms of φ_M.

The fidelity of the model (11) is determined by how closely its local slope at ω_C matches the slope (10) of the function (9) and its argument relates to phase margin (4).

To verify the accuracy of the model (11) we will calculate its magnitude, slope and phase at ω_C. By substituting $s = j \times \omega_C$ it is easy to find that for such function $|T(j\omega_C)| = 1$.

The slope of its magnitude at ω_C on a log-log scale is

$$\frac{d(\log|T(j\omega|)}{d(\log\omega)}|_{\omega=\omega_C} = \sin\varphi_M{}^2 - 2 \qquad (12)$$

By comparing (12) to the desired slope (10), one can find that the slope (12) exactly matches (10) at 0°, 45°, and 90°. The maximum discrepancy occurs at $\varphi_M \approx 22.5^o$ and 67.5°, where the slope error is less than 6% (approximately 1.2 dB/decade).

After some algebra, we find that the argument of the function (11) at ω_C is given as

$$\angle T(j\omega_C) = -180^o + \varphi_M \qquad (13)$$

Equations (12) and (13) demonstrate that the phase of the function (11) at ω_C is exactly related to φ_M (4), and the slope of its magnitude closely approximates the desired slope. This confirms that the function (11) can adequately approximate the generic behavior of a stabilized converter's open-loop transfer function near the crossover frequency, making this a tool for verifying control loop designs against time-domain specifications.

Transient Response Calculation

The step load response is obtained by substituting (11),

$$T(s) = \frac{\omega_C}{s} \times \frac{\sin\varphi_M \times s + \cos\varphi_M \times \omega_C}{s}$$

into (3),

$$\Delta V(s) \approx -\frac{\Delta I}{s} \times \frac{Rc+\frac{1}{sC}}{1+T(s)}$$

and applying the inverse Laplace transform.

After some mathematics one can find that the resulting solution takes three distinct analytical forms, depending on the sign of $(\cos(\varphi_M) - \sin(\varphi_M)^2/4)$. When $\cos(\varphi_M) - \sin(\varphi_M)^2/4 = 0$, which occurs at $\varphi_M \approx 76.345^o$, the response is critically damped. The response is underdamped for $\varphi_M < 76.345^o$ and overdamped for $\varphi_M > 76.345^o$.

Remarkably, the critical phase margin, obtained here independently of converter topology within the adopted model assumptions, coincides with the optimal value of 76° reported in reference [2] for a buck converter.

The main practical interest here concerns the underdamped region. In this region by applying the inverse Laplace transform to ΔV(s) one can find the transient recovery of the output voltage V(t):

$$V(t) = -\Delta I \cdot e^{-\sigma t} \times \left[Rc \cdot \cos(\omega_D t) + \frac{1-\sigma \cdot C \cdot Rc}{C \cdot \omega_D} \times \sin(\omega_D t)\right] \quad (14)$$

where $\omega_D = \omega_C \times \sqrt{(\cos\varphi_M - \frac{1}{4}\sin\varphi_M{}^2)}$ is the oscillation frequency, $\sigma = \frac{\omega_C \times \sin\varphi_M}{2}$ is the decay constant, and $\varphi_M <$ 76.345°.

The designer can use the properties of the feedback loop and output capacitance as "knobs" to achieve the desired load transient response.

As an illustration we will consider the following example:

- Load step ΔI = 5 A
- Output capacitance C = 220 μF
- R_c = 0.02 Ω (ESR)
- Crossover frequency F_C = 20 kHz and $\omega_C = 2\pi F_C \approx 125664$ radians/sec.

Fig. 1 provides transient response waveforms calculated from (14) for these conditions.

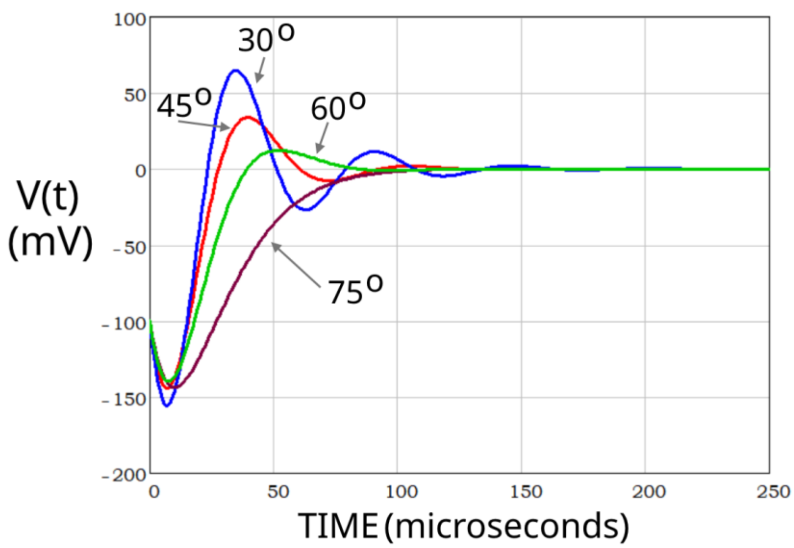

Fig. 1. Step load transient response (in millivolts) at various phase margins.

The initial voltage step is caused by the ESR as expected. The curves of Fig. 1 show that the phase margin mostly affects the recovery shape. Time to peak t_{PK} and peak deviation V_{PK} are relatively insensitive to phase margin. These properties depend primarily on the bandwidth ω_C and the capacitance C.

To validate the proposed bandwidth- and phase-margin–based model (11) we simulated the transient response of a 5-V buck converter by using an LTspice model.[5] The component values in the compensation circuit were automatically adjusted for various phase margins at the given crossover frequency. The resulting phase margins were verified with ac analysis. The simulation was done under the same conditions as in our above example.

Fig. 2 shows the resulting Bode plots of the simulation model.

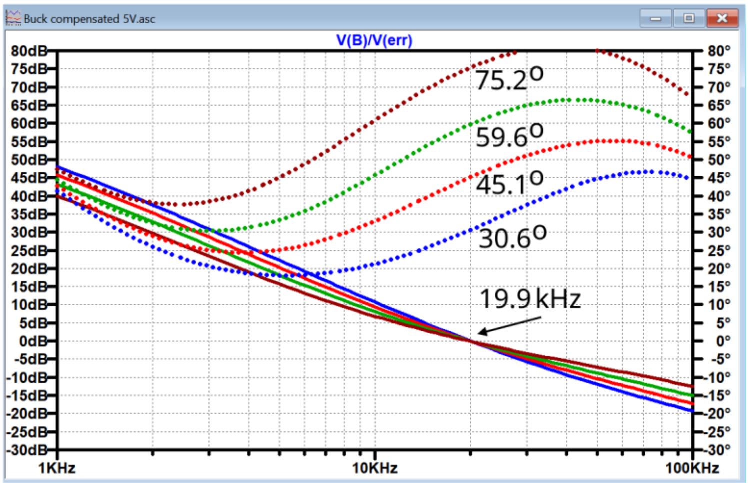

Fig. 2. Bode plots of the simulated buck converter.

Fig. 3 shows the simulated transient response for the same conditions as used in our example.

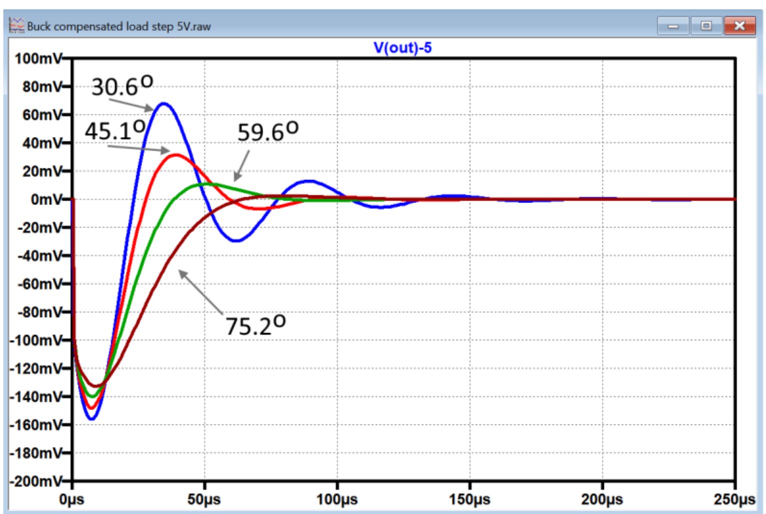

Fig. 3. Simulated step load voltage transient response at various phase margins.

By comparing Fig. 1 and Fig. 3 one can see that the simulation results confirm that the general dependence of the recovery waveform on phase margin follows the analytical derivation. In particular, it is seen that phase margin close to 76° yields practically aperiodic recovery as predicted above.

Calculation Of Transient Response Characteristics

Equation (14) allows estimation of the various characteristics of the transient voltage, such as settling time and peak overshoot in terms of the loop properties ω_C, φ_M and output capacitor (C and R_c).

Settling time t_S is defined here as the time from the initial onset of the load transient to the time when the envelope of |V(t)| returns to a specified fraction "ε" of the nominal steady-state value V_o:

$$|V(t_S)| \le \varepsilon \cdot Vo \;\; \exists\, t > t_{PK} \qquad (15)$$

The settling time in general is

$$t_S = t_{PK} + t_D \qquad (16)$$

where t_D is the decay time (the additional time after the first peak t_{PK} before the decaying response remains within the specified voltage band).

As in the previous section, we will still consider underdamped conditions ($\varphi_M < 76.345^o$). For convenience in the calculations, (14),

$$V(t) = -\Delta I \cdot e^{-\sigma t} \times \left[Rc \cdot \cos(\omega_D t) + \frac{1 - \sigma \cdot C \cdot Rc}{C \cdot \omega_D} \times \sin(\omega_D t)\right]$$

can be rewritten in a compact form as

$$V(t) = -\Delta I \cdot Z_{EFF} \cdot e^{-\sigma t} \times \sin(\omega_D t + \Theta) \qquad (17)$$

where

$$\Theta = \mathrm{atan}\left(\frac{C \cdot Rc \cdot \omega_D}{1 - \sigma \cdot C \cdot Rc}\right) \qquad (18)$$

$$Z_{EFF} = \sqrt{Rc^2 + \left(\frac{1 - \sigma \cdot C \cdot Rc}{C \cdot \omega_D}\right)^2} \qquad (19)$$

The Z_{EFF} here appears as an effective transient impedance, and Θ as a transient phase angle.

By setting $dV(t)/dt = 0$ and solving for "t" after some manipulations we find time to peak as

$$t_{PK} = \frac{1}{\omega_D} \times \mathrm{atan}\left(\frac{\omega_D}{\sigma} - \Theta\right) \qquad (20)$$

The absolute value of the peak deviation V_{PK} is obtained by substituting (20) into (17). After some simplifications,

$$V_{PK} = \Delta I \cdot Z_{EFF} \cdot \frac{\omega_D}{\omega_C \sqrt{\cos \varphi_M}} \cdot e^{-\sigma t_{PK}} \qquad (21)$$

The decay time is the time from the peak to the time when the envelope of |V(t)| returns to values within the voltage band εVo:

$$t_D = \frac{1}{\sigma} \ln\left(\frac{V_{PK}}{\varepsilon Vo}\right) = \frac{1}{\sigma} \ln\left(\frac{\Delta I \cdot Z_{EFF} \cdot \omega_D}{\varepsilon Vo \cdot \omega_C \sqrt{\cos \varphi_M}}\right) - t_{PK} \qquad (22)$$

Then the settling time is

$$t_S = t_{PK} + t_D = \frac{1}{\sigma}\ln\left(\frac{\Delta I \cdot Z_{EFF} \cdot \omega_D}{\varepsilon Vo \cdot \omega_C \sqrt{\cos\varphi_M}}\right) \qquad (23)$$

It's interesting to note that t_{PK} is cancelled out in the equation for t_S. Also, note that the expression (23) is based on the decaying exponential envelope. Strictly speaking, the actual waveform can enter the band εVo earlier than the envelope does; therefore, this is a conservative analytical settling-time estimation.

Fig. 4 provides examples of settling time for Vo = 5 V for various "ε" under the same load step and output capacitance as in our example above.

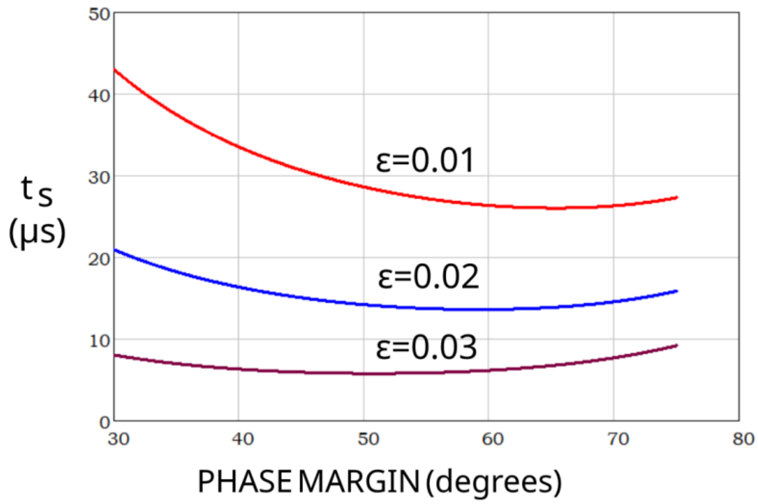

Fig. 4. Settling time (in microseconds) for various voltage thresholds.

One can see from Fig. 4 that a tighter settling requirement favors higher phase margin, whereas a more relaxed requirement needs lower one for faster settling. In our example, for 1% threshold (ε = 0.01) the optimal φ_M is about 65°, while for 3% (ε = 0.03) it's ~50°. There is no single "optimal" phase margin. It should be selected according to the required transient-response specification rather than a fixed 45°, which is sometimes perceived as the best compromise between stability and speed. The 45° can be viewed as the minimum acceptable value.

This naturally raises the question of whether a general expression for the optimum phase margin can be derived for a given settling-time requirement. To obtain a useful analytical approximation, we will assume an ideal output capacitor (R_c = 0). Then (23) becomes

$$t_S = \frac{1}{\sigma} \cdot \ln\frac{\Delta I}{C \cdot \omega_C \sqrt{\cos\varphi_M} \times \varepsilon Vo} \qquad (24)$$

Setting $dt_S/d\varphi_M = 0$ yields the following transcendental equation for the optimum phase margin:

$$\tan(\varphi_M) = 2 \cdot \ln\frac{\Delta I}{C \cdot \omega_C \times \varepsilon Vo} - \ln(\cos\varphi_M) \qquad (25)$$

Equation (25) has no closed-form solution, but the optimum phase margin can be obtained numerically by conventional engineering tools for the specific converter parameters and settling criterion of interest.

To estimate peak overshoot (i.e. the peak voltage after it reverses polarity) we note that the time span between the peak deviation t_{PK} and the subsequent polarity-reversed peak is half of the period of the oscillating frequency ω_D:

$$t_{OS} - t_{PK} = \frac{\pi}{\omega_D} \tag{26}$$

Therefore, the overshoot magnitude,

$$V_{OS} = V_{PK} \times e^{-\sigma\pi/\omega_D} \tag{27}$$

where V_{PK} is given by (21).

Equations (21), (23) and (27) can be used to select the values of output capacitance and bandwidth to meet application requirements.

The preceding analysis is based on the response to step load where transition mathematically occurs instantly. In practice, of course, load transitions occur over some non-zero time. With a finite slew rate di/dt assuming load change is still much faster than the response time, the actual time to peak is given, per reference [6] as

$$t_{PK}' = t_{PK} - \frac{\Delta I}{di/dt} \tag{28}$$

As the result, the peak deviation, settling time and overshoot values are reduced accordingly.

Conclusion

This article analyzes the generic relationship between feedback loop characteristics and load transient response. The proposed approach avoids detailed circuit-level derivations by treating the loop as a behavioral system described by its phase margin, crossover frequency (bandwidth), and output capacitance. This formulation is intentionally simplified to emphasize analytical clarity and direct insight into dominant design tradeoffs.

A generic, topology-independent representation of the loop gain T(s) in the vicinity of the crossover frequency is constructed, enabling a simple direct mapping between frequency-domain design parameters and time-domain transient behavior. Closed-form expressions for the transient voltage waveform, peak deviation, settling time and overshoot are derived. It was found particularly that phase margin close to 76˚ yields aperiodic recovery.

The results provide a practical means for establishing design targets that meet output voltage transient requirements, without need of the full circuit-level analysis. The derived expressions are straightforward to implement in common engineering tools such as Mathcad, MATLAB or Excel, facilitating quick evaluation during the design process.

Owing to the simplifying assumptions used in the analysis, the obtained expressions are intended for quick estimation of transient behavior. Consequently, the derived expressions should be interpreted as first-order analytical approximations intended to capture the transient behavior and provide design insight, rather than exact quantitative predictions.

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About The Author

Lazar Rozenblat is a retired electronics engineer who has been involved in power supply design for over 35 years. Prior to his retirement, Lazar was a principal applications engineer at Microchip Technologies and Microsemi in Melville, New York, principal engineer at TDI Long Island R&D Center in Ronkonkoma, New York, and senior engineer at Condor DC/Todd Products in Brentwood, New York. He received an MSEE from Leningrad (now Saint Petersburg) Electrotechnical Institute of Communications in 1972.

For more on transient response in power supply design, see How2Power's [Design Guide](), locate the "Design area" category and select "[Transient Response]()".