

Measuring Parasitic Inductance Of Sense Resistors

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Sense resistors sense current and output voltage, simply by Ohm's Law. They are typically low in resistance to avoid excessive power dissipation. They are also typically found in circuits with dynamic activity such as comparator inputs. Ideally, they are resistive, but parasitic series inductance can cause problems. This article explains how the small inductance, mainly contributed by resistor or circuit-board trace leads, can be significant and how to measure it.

Two methods of measuring parasitic inductance are explained here. The first is a frequency-response method using a sine-wave source generator and oscilloscope. While it's convenient, it has drawbacks in terms of accuracy. A second approach, a time-domain measurement using a pulse generator and scope overcomes the limitations of the first method. Both methods are performed here with results compared against a calculation of parasitic inductance.

Why It Matters

A sense resistor has not only resistance. So while there are times you may model it as an ideal component, a better model includes parasitic series inductance. Terminal leads (or terminal traces for surface-mount resistors) contribute series inductance. In most resistor applications this inductance is of no consequence for it forms a time-constant that is very small. But when resistance is also small in value the time-constant $\tau = L/R$ becomes large enough so that the frequency $1/\tau$ can affect the loop dynamics of the power circuit.

It is not uncommon to encounter parasitic inductances in the 50- to 200-nH range. This is too small in value to measure accurately on mainstream RLC meters but can be measured conveniently on the lab bench by time- and frequency-domain methods. A 100-m Ω sense resistor with 100-nH series inductance has a time-constant of 1 μ s. And a 10-m Ω resistor will have a time constant of 10 μ s, or a break frequency of about 16 kHz, within the bandwidth of many power-circuit feedback loops.

The series RL combination has an impedance of

$$Z_s = s \cdot L_s + R_s = R_s \cdot \left(s \cdot \frac{L_s}{R_s} + 1 \right) = R_s \cdot (s \cdot \tau_s + 1)$$

where $s = j \cdot \omega = j \cdot 2 \cdot \pi \cdot f$ for frequency-response analysis. The additional parasitic inductance adds a zero at frequency $\omega_z = 1/\tau_s$ in the feedback path. Where current-mode control is implemented, this added zero becomes an additional pole in a current-amplifier feedback loop. Consequently, the zero cannot be ignored and an estimate of the parasitic inductance becomes worthwhile.

Methods for series-inductance measurement are based on the frequency and time domains. The test circuit shown in Fig. 1 can be easily built on a lab bench for measuring the parasitic series L_s .

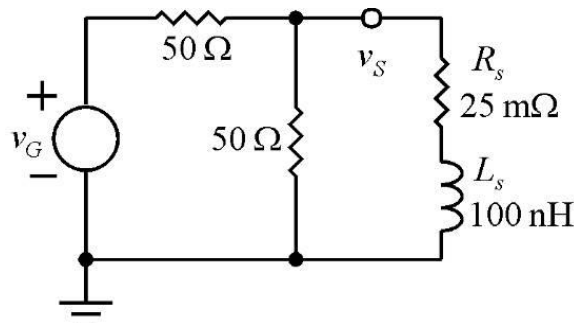


Fig. 1. Test circuit for measuring sense resistor parasitic inductance.

Typical function generators have 50-Ω outputs and can be applied to this measurement; the center 50-Ω terminates the source 50 Ω. Combine generator and terminating source resistance to form the equivalent series source resistance of $R_g = 25 \Omega$. The transfer function of this circuit is thus

$$\frac{v_s(s)}{i_g(s)} = \frac{v_s(s)}{v_g(s)/R_g} = \frac{R_g}{2} \cdot \frac{sL_s + R_s}{(sL_s + R_s) + R_g} = \frac{1}{2} \cdot (R_s \parallel R_g) \cdot \frac{s \cdot \frac{L_s}{R_s} + 1}{s \cdot \frac{L_s}{R_s + R_g} + 1}$$

For $R_g \gg R_s$, this is approximated by

$$\frac{v_s}{i_g} \approx \frac{1}{2} \cdot R_s \cdot \left(s \cdot \frac{L_s}{R_s} + 1 \right), R_g \gg R_s$$

Then by choosing f_a such that the inductive reactance is much larger than R_s ,

$$\frac{v_s}{i_g} \approx \left(s \cdot \frac{L_s}{R_s} \right), R_g \gg R_s, \omega_a \cdot L_s \gg R_s$$

By frequency-sweeping the generator to a higher frequency f_a the amplitude increases by a times. Substituting $s = 2 \cdot \pi \cdot f_a$ and solving,

$$L_s = \frac{a \cdot R_s}{2 \cdot \pi \cdot f_a}, a \gg 1$$

(Use a value of $a \gg 1$ to avoid the knee of the frequency-response curve around the break frequency.) For $a = 5$, sweep the generator upward in frequency until the measured amplitude is 5 times that of its unchanging, low-frequency value. Substitute a and f_a into the above equation for L_s .

Two factors limit the usefulness of this method. First, resistance (distinct from reactance) increases with frequency from the *skin effect*, the self-induced eddy-current effect within a conductor. Above the frequency f_δ at which the skin depth of current penetration into the conductive material equals the conductive wire radius, resistance increases. Thin-film resistors usually show this effect at higher frequencies than bulk resistors.

Second, if not terminated in 50 Ω the source cable will have standing waves which degrade amplitude measurement accuracy. If possible, place R_s at the generator output terminals, shown below in Fig. 2a. In this photo, a ×10 oscilloscope probe with ground clip is shown. At the center (Fig. 2b) is a sine-wave source driving R_s from a 50-Ω output into a 50-Ω coaxial line to a scope with a 50-Ω line terminator, shown on the right (Fig. 2c).

Especially for chip resistors, a General Radio (GR) line insertion unit preserves the 50-Ω cable impedance along the line. The generator is terminated in its source resistance (50 Ω) at the end of the cable. The oscilloscope “probe” is a 50-Ω coaxial cable terminated at the scope CH 1 input in a 50-Ω BNC terminator. This results in an equivalent source resistance of 25 Ω—still 1000 times that of a 25-mΩ sense resistor.



(a)



(b)



(c)

Fig. 2. Half-loop manganin sense resistor connected to the function generator at its output with voltage sensing across it (a) by an oscilloscope (OS) probe with ground clip connected to the ground side of the resistor and to a 50-Ω transmission line (b) terminated in a CH 1 50-Ω OS input (c).

The test setup shown above using the 25-mΩ Manganin resistor resulted in a $\times 3$ increase in amplitude at 550 kHz. That calculates to be a value of $L_s = 22$ nH, a value also measured by an RLC meter at 1-kHz. (It is also about 25 nH based on geometry.) It is reasonable to expect the resistance of R_s to be about 27.5 times larger, or 0.688 Ω at 550 kHz, than at 1 kHz.

This method produced a reasonable estimate of inductance in this particular example because a low measurement frequency was adequate. For cases where a higher frequency must be used, there may be additional error due to skin effect. However, in such instances, by accounting for skin effect, the accuracy of the frequency method can be improved.

Skin-effect resistance is not a reactance but is nevertheless a *dynamic* resistance, one that is frequency-dependent. As a resistance it does not affect the phase of the source current. The skin depth at 80°C for Cu and Al simplify to the following formulas when the materials parameters are substituted for resistivity and permeability:

$$\delta_{Cu} = \sqrt{\frac{2.131 \mu\Omega \cdot \text{cm}}{(1.257 \mu\text{H/m}) \cdot \pi \cdot f}} = \frac{73.46 \text{ mm}}{\sqrt{f/\text{Hz}}} \approx \frac{73.5 \text{ mm}}{\sqrt{f/\text{Hz}}}, \text{ Cu}$$

$$\delta_{Al} = \sqrt{\frac{3.487 \mu\Omega \cdot \text{cm}}{(1.257 \mu\text{H/m}) \cdot \pi \cdot f}} = \frac{93.97 \text{ mm}}{\sqrt{f/\text{Hz}}} \approx \frac{94 \text{ mm}}{\sqrt{f/\text{Hz}}}, \text{ Al}$$

At the same frequency, Al skin depth is greater than Cu though its static resistivity is higher. For etched-circuit-boards (ECBs) a 1 oz/ft² ($h = 34.3 \mu\text{m} \approx 35 \mu\text{m}$ thick) board trace has δ equal to the plating thickness ($\delta = h$) at 60°C at a frequency of $f_\delta = 4.3$ MHz.

The skin effect in a round wire of radius r_c is expressed as the round-wire conductor radius in units of skin depth;

$$\xi_r = \frac{r_c}{\delta}$$

Because of the skin effect, wire size for wire sense resistors cannot simply be chosen for ampacity. Instead, multiple strands of smaller wire with the combined ampacity of one larger wire each have a smaller ξ_r , and conductive utilization of their area is effectively increased. The total current flows at an equivalent depth of δ .

If the wire radius is chosen to be $r_c = \delta$, or $\xi_r = 1$, its conductive area $\approx 63\%$ at the given frequency of current.

Because current distribution is exponential with a tail going to infinity beyond a finite r_c the current density is not exactly uniform though $\xi_r = 1$ is an approximate optimization criterion for isolated (single) wires.

The *skin-depth frequency* f_δ at which $r_c = \delta$ is found by solving a skin-depth equation:

$$f_\delta = \left(\frac{73.5 \text{ mm}}{r_c} \right)^2 \cdot \text{Hz, Cu, } 80^\circ\text{C}$$

$$f_\delta = \left(\frac{94 \text{ mm}}{r_c} \right)^2 \cdot \text{Hz, Al, } 80^\circ\text{C}$$

A #18 AWG Cu wire has $r_c = 0.516 \text{ mm}$ and $f_\delta = 20.6 \text{ kHz}$ at 80°C . (See in *Power Magnetics Design Optimization*, the section, "Skin Effect" for more detail, as linked to in the reference).

Despite its plausibility, this frequency-response measurement technique is of limited applicability. The above example measured with an RLC meter did so at 1 kHz at which frequency there is little eddy-current effect. This shows that the frequency-response scheme works under the condition that the test frequency is low enough. A frequency sweep with the high frequency well below f_δ avoids the skin-effect problem but might be too low to make any kind of reliable measurements. At higher frequencies, the voltage dropped across the inductance is higher, thus more measurable, but skin-effect might invalidate the measurement.

The value of L_s in the above formula depends on R_s which increases with frequency for skin depth less than the conductor radius or thickness. For 18-gauge wire, resistance increases with frequency above $f_\delta = 20 \text{ kHz}$. For thin-film resistors, which are becoming more common, the technique has some merit but is still affected by R_s variation with frequency.

The Pulse-Response Method

A time-domain-oriented method of measuring sense-resistor parasitic inductance is the pulse response method using a pulse generator. In the time domain R_s can be eliminated from its effect on the measurement—a big advantage over the frequency method.

Using the same setup as before except with a pulse generator as a terminated source, the generator voltage pulse rising transition is adjusted to be in this example 10 V in 50 ns, or 200 V/ μs . Then the source is still considered to be a current source (because $R_g \gg R_s$) that drives the sense resistor with a current ramp of

$$\frac{\left(\frac{dv_g}{dt} \right)}{R_g} = \frac{(200 \text{ V}/\mu\text{s})/2}{25 \Omega} = 4 \text{ A}/\mu\text{s}$$

(The /2 voltage attenuation is from the 50- Ω termination divider.) The inductance follows from the $v-i$ relationship for inductance;

$$L_s = \frac{v_L}{\left(\frac{di}{dt} \right)}$$

where v_L is the constant inductor voltage response to a current ramp.

When driven by a Tektronix PG508 pulse generator the Fig. 3 waveforms were observed.

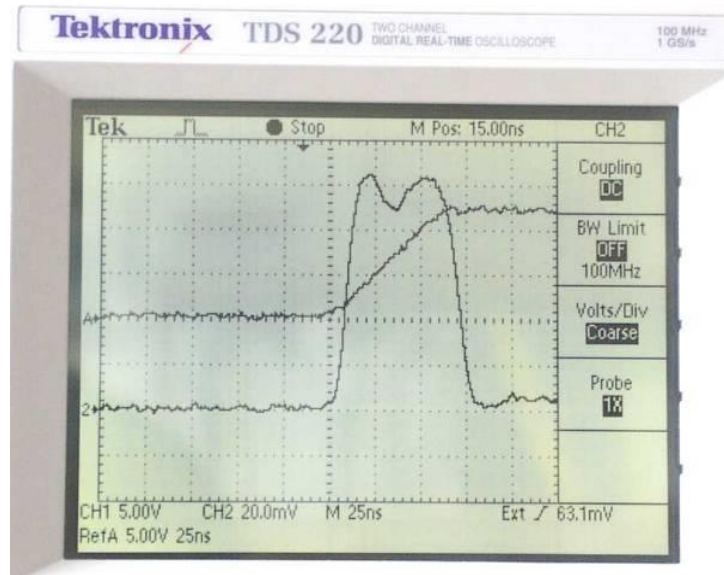


Fig. 3. Oscilloscope screen display of trace A: the PG source waveform; and trace 2: the differentiated result.

The upper waveform (A) in Fig. 3 is the open-source voltage without sense resistor; the channel 2 waveform is the response with resistor, though only approximately time-aligned with the (stored) A waveform. (Note the $\times 1$ probe setting for channel 2 because it is 50- Ω terminated.)

The voltage v_s steps up to v_L , a value of about 100 mV (channel 2, 20 mV/div). The current-ramp voltage drop across R_s causes a ramp-up superimposed on v_L which is negligible and indiscernible amidst waveform ringing. The pulse flattens on top to a constant voltage, leaving a constant-current drive of R_s producing about 6 mV ($\frac{1}{4}$ div). The value of R_s can be calculated from the voltage-divider formula, where $V_g = 12.5$ V drives $R_g = 25$ Ω in series with R_s to produce about 6 mV across it;

$$R_s = \frac{R_g}{\frac{V_g}{2 \cdot V_s} - 1}$$

For this measurement, the resulting value of R_s is about 24 m Ω , or 4% low from the 25 m Ω measured with an RLC meter.

The inductance is calculated from the measured voltage to be

$$L_s = \frac{100 \text{ mV}}{4 \text{ A}/\mu\text{s}} = 25 \text{ nH}$$

As noted previously, this value can be expected based on geometry.

Measurement of the series inductance of small-value resistors is difficult. A useful approximate value can be measured, however, with this simple setup of a pulse generator and an oscilloscope. This method results in the small values not measurable with mainstream-performance RLC meters. An impedance analyzer could provide more accurate measurements but at a much higher price.

Reference

[Power Magnetics Design Optimization](#) by Dennis Feucht.

About The Author



Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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